

Minimizing the variance of the coverage ratio as an approach to optimize the exchange rate risk of Brent futures contracts

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Abstract. Derivatives markets show that their structure is always characterized by periods of strong price fluctuations. This is true regardless of the underlying asset of the futures contracts considered, whether they are commodities, interest rates, exchange rates, shares, stock market indices, etc. By locking in future prices, the primary objective of these markets is to limit the risks faced by operators. This article proposes a new method of optimizing the coverage ratio by futures contracts to minimize price variance and thus apply this new technique to reduce the risk associated with Brent price volatility for the period from January 2010 to December 2020. The variance minimization model of Ederington's (1979) is the first and most widely used coverage model and the one that dominates the literature on this area which helps to find the optimal coverage ratio, and is also the objective function in our particle swarm optimization algorithm in MATLAB and we will better interpret our results with statistical analysis and lastly, we will evaluate the effectiveness of the coverage model.

Keywords: Brent oil / risk / price volatility / coverage ratio / optimization / particle swarm optimization

1 Introduction

Price volatility, compounded by geopolitical tensions, increases the uncertainty surrounding oil price trends. Price spikes are not uncommon in the oil market, and to some extent they reflect a gradual increase in daily price volatility. In this sense, speculators trade oil futures to profit from rising or falling prices. These traders do not take any risk on the physical product, but they usually have substantial commitments in the spot or futures markets and trade in futures contracts to minimize their risk to price fluctuations. Although the positions held by these operators represent a relatively small proportion of the total futures contracts traded, the corresponding net positions can be significant and their sudden change is likely to exert a strong influence on prices from time to time [1–4].

Futures contracts allow effective risk hedging strategies to be set up due to the correlation between spot and forward rates. Thus, if the risk becomes effective, losses realized in the spot market will be offset by gains in the futures market. The coverage ratio is a function of the sensitivity of spot and forward prices. Setting up a hedging strategy requires first calculating the number of contracts to buy or sell. Perfect hedging is achieved if the futures and spot prices are perfectly correlated. But generally, this is

not the case, which involves estimating the relationship between possible price movements. This generates a basis risk between these two markets which is the main limit in the implementation of hedging strategies [5–7].

This is the risk linked to the difference in sensitivity to market fluctuations between two financial instruments or between a hedging instrument and the hedged item. This risk reflects the fact that the hedging instrument does not change exactly as the covered item. We are looking for the coverage ratio that minimizes the variance of the residual coverage error. The advantage of variance is that it provides a linear rule for hedging a contract [8].

To solve this problem, we choose the PSO algorithm optimization invented in 1995, known as the most powerful and used metaheuristic for hard optimization problem, it's a high-resolution performance inspired by the simulation of the social behavior of the bird. The main objective of this article is to find the optimum of the coverage using the Ederington Model (1979) of the main function 'variance minimization model' to our portfolio that we are studying and also evaluate the efficacy of the coverage. This article explains in the first section an overview of the currency risk literature and the equations that we will use. The second section of this article is devoted to recall and describe PSO that will help us find the optimum. The third and last section is devoted to showing and analyze the performance results.

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2 Literature review

Ederington (1979), a pioneer of econometric approaches, demonstrated that the optimal coverage ratio is only the slope of a linear regression by the least squares method where the forward price is the explanatory variable and the spot price is the dependent variable. Several more recent studies have gone beyond this simplifying approach. The most interesting are those incorporating the sensitivity of prices to the arrival of new market information. Stoll and Whaley (1990) applied an “autoregressive vector” (VAR) model to modulate prices in the spot and futures markets.

The major criticism leveled at this family of models is the ignorance of the cointegration hypothesis between the two forward and spot series. Gosh (1993) and Lien (1996) demonstrate that the existence of a cointegrating relationship between forward and spot prices can lead to a biased coverage ratio if the error-corrected term is not considered in the estimate. This term is measured by the instantaneous difference between futures and spot prices; however, it represents the basis between the two markets. Several empirical studies, including that of Yang (2001), conclude that the coverage ratio estimated by the error-corrected vector model (VECM), which takes into account the basis between the two markets, is more efficient than that estimated from the VAR model to hedge the interest rate risk [9–11].

All these regression models consider that the covariance and the variances of Forward and spot yields are constant. This leads to a constant hedge ratio and consequently to a static hedge strategy. This stability of variances and covariances is always called into question by analyzes of time series. Most empirical studies express a heteroskedasticity effect of market price data. This prompts us to look at a dynamic hedge ratio that allows dynamic variances to be considered, taking into account such a heteroscedasticity assumption.

2.1 Hedging by futures contracts

At the same time, academics have taken a particular interest in the concept of hedging by futures contracts, which has been the subject of much research, which essentially revolves around three central axes which are: (i) identification of the objective function of the hedge and the analytical derivation of the optimal ratio (ii) the proposal of models for empirical estimation of this ratio (iii) the evaluation of the effectiveness of the hedge.

Generally, any hedging transaction aims to protect an existing (or expected) position in the spot market against an unanticipated price change. The intuitive idea behind a hedging transaction is that the position at term produces a gain when the position held in the cash market experiences a loss, and vice versa. In the presence of residual risks, the coverage becomes imperfect. This is referred to as optimal coverage. Its main objective is to minimize the dispersion of the value of the hedged position [12–13].

The optimal coverage ratio can be estimated by the regression method. This consists of regressing the change in

the price of the futures contract on the change in the spot price of the asset to be hedged.

2.2 Variance minimization model

In 1960 the scientific Johnson who was the first one to observe the important to coverage and reduce the currency risk in the international trade market and the one who created that was published by Ederington (1979). The Ederington model was based by the portfolio theory and he found the following equations of the return variance $\text{Var}(P)$ (see the formula (1)) and the return expectation $E(P)$ (see the formula (2)) formed by the spot and forward position which is the constraint to our variance minimization equation as shown in the following formula [14–17]:

$$\text{Var}(P) = X_s^2 \cdot \sigma_s^2 + X_f^2 \cdot \sigma_f^2 + 2X_s \cdot X_f \cdot \sigma_{s,f} \quad (1)$$

X_s : spot proportion assets (holdings) in the portfolio;
 X_f : forward proportion assets (holdings) in the portfolio;
 σ_s : variance of the spot price variation over a given period;
 σ_f : variance of the forward price variation over a given period;
 $\sigma_{s,f}$: covariance of the two-price variation over a given period.

$$E(P) = X_s E(\Delta P_s) + X_f E(\Delta P_f) \quad (2)$$

ΔP_s : spot price variation over a given period; ΔP_f : forward price variation over a given period.

After having rewritten the equation according to the coverage ratio by wanting to find the optimal ratio by minimizing the risk under the constraint, and by applying several operations they got the formula (3) below of the optimal coverage ratio (EOCR) that we will use to compare with our coverage ratio obtained by the PSO program (POCR):

$$b_E^* = \frac{\sigma_{s,f}}{\sigma_s^2}. \quad (3)$$

3 Particle swarm optimization algorithm

The Particle Swarm Optimization (PSO) is a well-known meta-heuristic that was developed by two scientists, sociopsychologist James Kennedy and electrical engineer Russel Eberhart in 1995 for solving continuous variable problems. This algorithm was modeled based on social behavior or for more precision on social interactions between “agents” called “particles” and which represents a “swarm”.

The particle swarm optimization optimizes the problems using iteration of the candidate solution, then we change the position and speed and check how close we are to solve the problem and know the best solution, then update the other particle as we optimize the problem, this algorithm can search for a large space of candidate solutions [18–21].

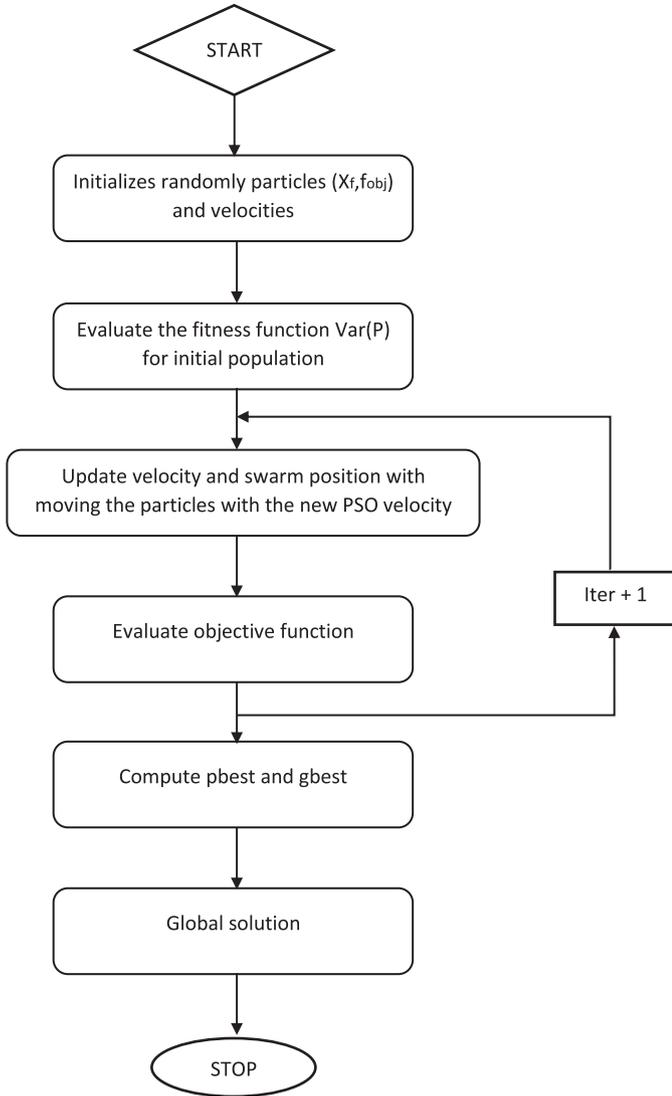


Fig. 1. PSO flowchart.

First, the algorithm begins by generating initial particles that are in our case the coverage ratio with their speeds at random in order to initialize a population in our area of research. Then, inside a loop which is repeated until convergence, we determine the best global position of the particle that are close to the level of risk that we want which we named f_{obj} , and in our case we took it equal to zero using the Hill & Schneeweis equation (1982) who took the same Ederington model (1979) and define the main minimization objective function that we will use on our application case [22–24]. The spot position is actually fixed that why we don't find it in the following function. the following equation which is the objective function of our program that explained in the previous part:

$$\begin{cases} \text{Min Var}(P) = \sigma_s^2 + X_f^2 \cdot \sigma_f^2 + 2 \cdot X_f \cdot \sigma_{s,f} \\ \text{Constrained with :} \\ E(P) = E(\Delta P_s) + X_f \cdot E(\Delta P_f) \end{cases} \quad (4)$$

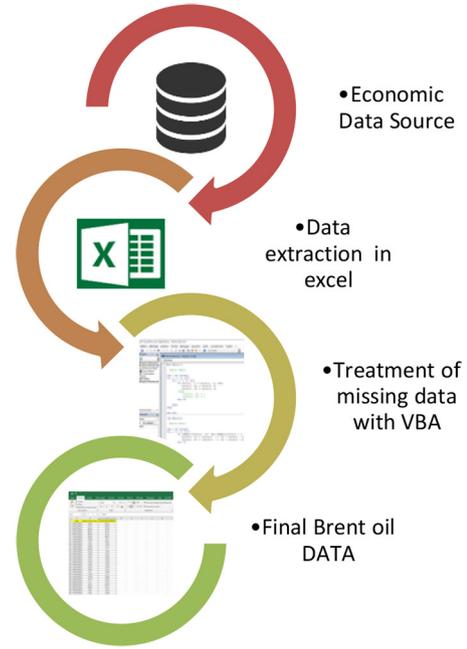


Fig. 2. Treatment of missing data process.

In each loop we move the particle to a new position X_{t+1} by calculating the speed V_{t+1} according to the following equations [25–27]:

$$\begin{aligned} V_{t+1} = C \times (V_t + C_1 \cdot r_1 (X_{pbest} - X_t) + \\ tC_2 \cdot r_2 \cdot (X_{best_{vois}} - X_t) \\ + C_3 \cdot r_3 (X_{best_{glob}} - X_t) \end{aligned} \quad (5)$$

C : construction coefficient; C_i^o : social parameter; r_i : random number between 0 and 1; X_t : current position; X_{pbest} : best position of the particle; $X_{best_{vois}}$: best neighborhood position; $X_{best_{glob}}$: best overall position.

And the equation below shows us how to generate the new position of each particle:

$$X_{t+1} = X_t + V_{t+1}. \quad (6)$$

So, we can conclude that the movement of the particle I between t and $t+1$ depends on the speed which is influenced by three parameters: the best global position the particle that overcome all the population, the best neighborhood position because in our program we have divided the population into equal groups and finally the best position of the particle she's ever been through using her memory. And below a flowchart (Fig. 1) which summarizes the process followed by PSO program.

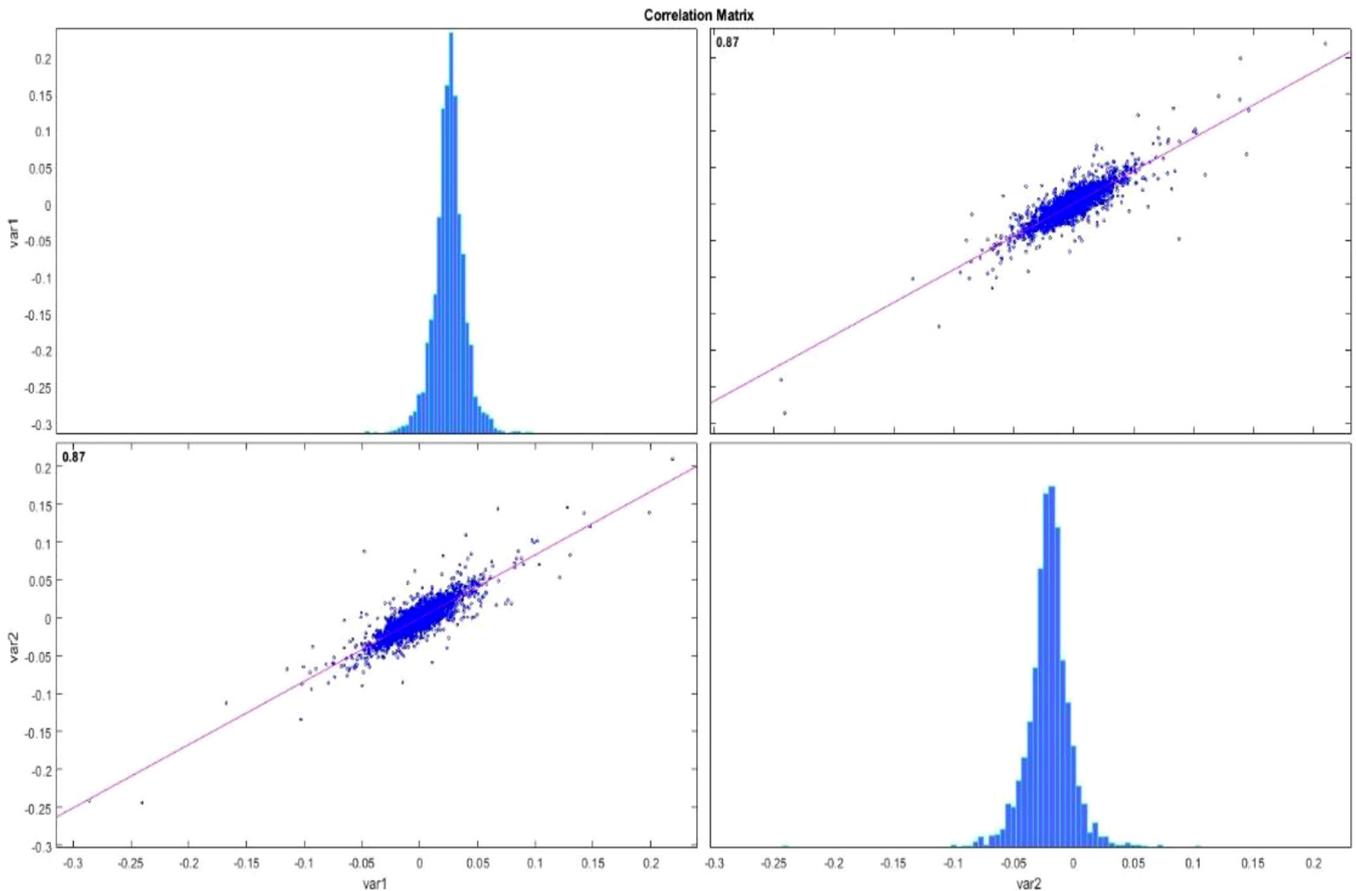
4 Data treatment

4.1 Case study description

Oil is a basic good whose price is determined by supply and demand in the “spot market”. In addition, operations are

Table 1. Descriptive analysis of spot and forward data.

	S	F
Average	2.005×10^{-4}	1.191×10^{-4}
standard deviation	0.023	0.022
Kurtosis	23.874	19.834
Skewness	-0.4196	-0.2738
Jarque-Bera	8652.704	5624.344
Q	43.709	35.867
Observation number	2855	2855

**Fig. 3.** Correlation matrix.

carried out on futures markets to allow producers to forward quantities of oil at a price fixed in advance, and thus to protect themselves against any unfavorable price variations. The strong growth in oil transactions led to greater price volatility. In times of crisis, the price of a barrel has thus been able to deviate excessively, both upwards and downwards, from the fundamentals of the oil market.

For the data used, Brent prices are spot prices (quantities purchased for immediate delivery, which differs from the quotation of futures contracts). These are benchmark prices for the market. However, some commodities are traded on the basis of futures contracts. The prices are quoted in US dollars. These are the daily closing session prices that cover the period analyzed. These

prices are taken from the Bloomberg platform and cover the period from January 4, 2010 to December 22, 2020 except for the week of 18/11/2019. After processing the data as shown in Figure 2, the final sample that we will use comprises 2855 daily observations.

Figure 3 represents the correlation matrix of the spot and forward crude oil data which contains a significant correlation coefficient between the two variables; therefore a strong linear relationship between these last two and tend to increase and decrease together, this figure also contains the distribution of each variable on the diagonal only with the statistical coefficients descriptive of the spot (S) and forward (F) data summarized on Table 1 we will have a clear idea about their distributions.

Table 2. Descriptive analysis of the coverage ratio.

	2 Weeks POCR	4 weeks EOCR	POCR	EOCR
Average	0.0910	-0.7404	-0.2007	-0.7207
Standard deviation	1.9814	0.2921	1.9757	0.1921
Kurtosis	1.0087	3.3373	1.0396	2.9921
Skewness	-0.0913	0.2942	0.1982	-0.1985
Jarque-Bera	7.9140	0.9104	3.9446	0.1555
Q	16.9432	76.1678	35.3695	179.8796
Observation number	285	285	142	142

The Skewness coefficient measures the degree of asymmetry of the distribution which in our case is negative and therefore a distribution shifted to the right of the median. We also note the leptokurtic form of the distributions of series (S) and (F) since the Kurtosis coefficient which measures the flattening of the distribution are of positive values.

The Jarque-Bera statistic is a test to determine if the data follow a normal distribution, they have a significant value which proves that the variables S and F rejects the null hypothesis that they do not follow the normal distribution at the default 5% significance level. We also applied the Ljung-Box (Q) statistical test that show if the data studied are affected by trend and seasonality and in our case both variables accept the null hypothesis that they are not autocorrelated with a critical value equal 31,41.

5 Results

5.1 Results interpretation

The risk analyzed is the risk linked to the difference in sensitivity to market fluctuations between two financial instruments or between a hedging instrument and the hedged item this risk reflects the fact that the hedging instrument does not change exactly as the covered item.

The objective of this work is to determine the optimal hedge ratio, i.e., the proportion of forward contracts to be held in a hedged portfolio made up of a given quantity of spot asset and a proportion of futures contracts to be held in the futures market. We are looking for the coverage ratio that minimizes the variance of the residual coverage error therefore minimize the risk. The advantage of variance is that it provides a linear rule for hedging a contract.

Table 2 represents the statistical description of the results of the coverage ratio obtained by the PSO program (POCR) and the Ederington optimal coverage ratio (EOCR) both according to the model of minimization of the variance of the Ederington model using the equation explained in the first part. The skewness coefficient is both negative for the POCR with 2 weeks frequency and EOCR with 4 weeks frequency, therefore the distribution is shifted to the right of the median and left for the others because their value are positives. We also note the leptokurtic form of the distributions of both POCR and EOCR with 2 weeks and 4 weeks frequency since the Kurtosis positive. We also applied the Jarque-Bera statistic test on our results and we

concluded that only the POCR with 2 weeks and 4 weeks frequency rejects the null hypothesis so POCR does not follow the normal distribution. Also, we can notice that all the Ljung-Box (Q) statistical value except POCR with 2 weeks frequency are bigger than the critical value so that means that they are not affected by time.

5.2 Measuring the effectiveness of a hedge

A hedging strategy is said to be effective if it succeeds in minimizing the variance of the gain of the hedged portfolio. If the hedger negotiates a number of contracts according to all the rules set out above, we are entitled to believe that the hedging strategy could achieve the objective it set for itself. In fact, this is not the case since the effectiveness of a hedging strategy depends closely on the degree of correlation observed between changes in the spot price of the exposed asset and those in the forward price.

5.2.1 Results interpretation

In Figures 4 and 5 we have the evolution of the variance which illustrates the rate of risk reduction that can be obtained using the POCR and EOCR on the different frequencies (2 weeks and one month) and we notice that using the PSO program we were able to better optimize the risk compared to that of Ederington. As we can see, the evolution of the POCR variance is always smaller than the EOCR variance and the difference between them becomes more important as the frequency grows, we can also clearly see the gaps between the two become larger on the one-month frequency than the 2 weeks frequency.

In Figures 6 and 7 we have the evolution of the ratio obtained value from the Ederington optimal coverage and the PSO program, we can clearly notice that the POCR is less variant and turns around 0 than the EOCR which is more variant. We can also notice unlike the variance that the more the period of the frequency decreases, the more the values of the ratio approach.

The PSO program gives us the lowest value to have a small variance and therefore the lowest risk, but we have to change some parameters on the MATLAB program. We can thus have a better ratio than the normal ratio with minimal risk if we really want to invest in Brent futures contracts unlike the normal ratio which will be more or less stable. For a monthly frequency, there is a minimum risk for the ratio resulting from the PSO program which

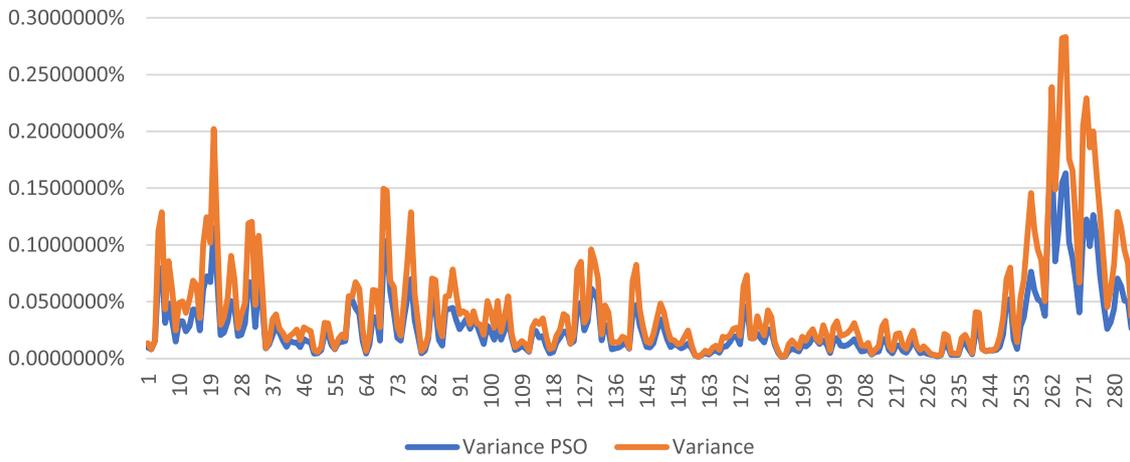


Fig. 4. Evolution of the variance over the 2-week hedging horizons.

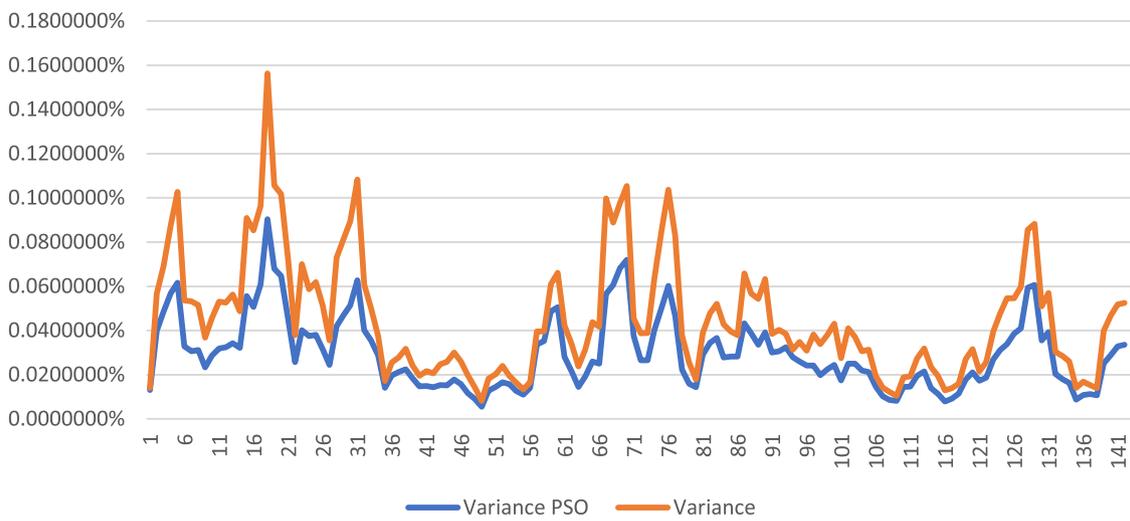


Fig. 5. Evolution of the variance over the one-month hedging horizons.

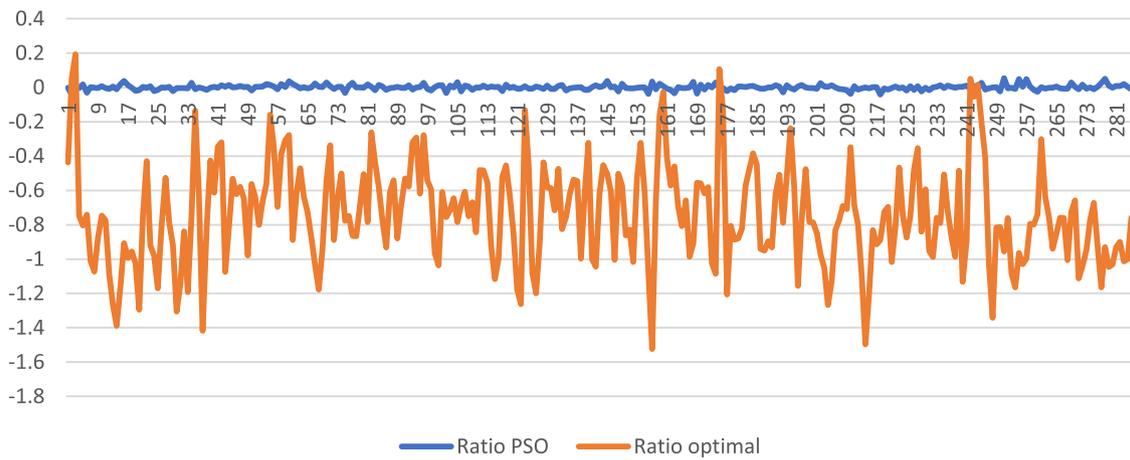


Fig. 6. Evolution of the optimal ratio over the 2-week hedging horizons.

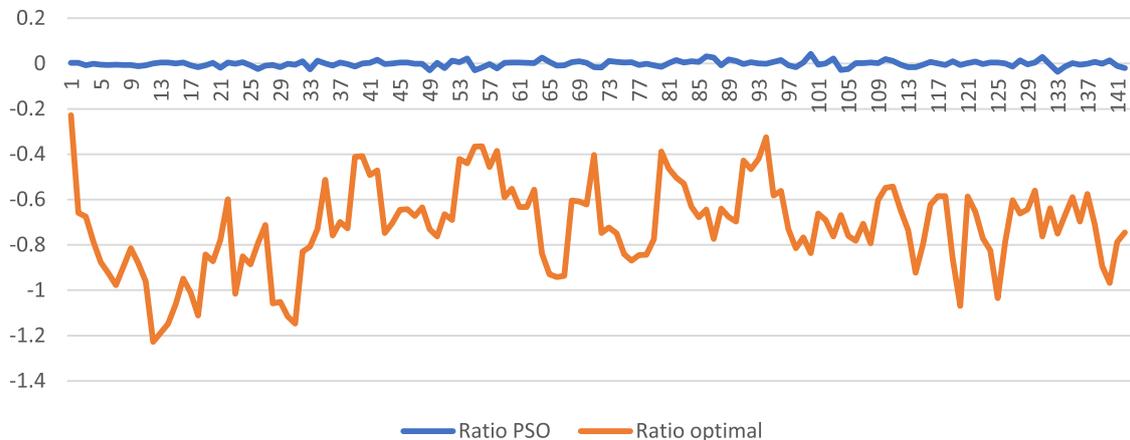


Fig. 7. Evolution of the optimal ratio over the one-month hedging horizons.

remains more stable and which varies around 0, unlike the optimal normal ratio which is more variable and with a higher risk and whose values are more dispersed those of the PSO algorithm. Applying the PSO algorithm on the objective function of variance minimization allows us to conclude that the value of the ratio from the PSO program is less variant and more stable than the optimal normal ratio.

6 Conclusion

All hedging strategies reduce volatility. Hedging provides the best indicators of risk when futures contracts are optimally weighted. It varies little according to the periodicity used, and the risk indicators cannot always be improved by the simple increase in the weight of futures contracts. This paper proposes a much faster and better way to find the best optimum to minimize the risk of hedging using the PSO algorithm that already known as the best metaheuristic and are easier to add and change parameter via the situation. it is a general way to measure the risk and find the most proper ratio that can have the minimum risk or have the risk that we want to not exceed. The PSO program is more effective in finding the best optimum under our own most favorable conditions as opposed to the optimal coverage ratio of Ederington.

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