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Investigation on Buckling of Orthotropic Circular and Annular Plates of Continuously Variable Thickness by Optimized Ritz Method

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Abstract – This paper investigates symmetrical buckling of orthotropic circular and annular plates of continuous variable thickness. Uniform compression loading is applied at the plate outer boundary. Thickness varies linearly along radial direction. Inner edge is free, while outer edge has different boundary conditions: clamped, simply and elastically restraint against rotation. The optimized Ritz method is applied for buckling analysis. In this method, a polynomial function that is based on static deformation of orthotropic circular plates in bending is used. Also, by employing an exponential parameter in deformation function, eigenvalue is minimized in respect to this parameter. The obtained results show that in plate with identical thickness, increasing of outer radius decreases the buckling load factor

Keywords: Buckling, Orthotropic circular and annular plates, Optimized Ritz, Variable thickness.

1 Introduction

Orthotropic circular and annular plates are always used by mechanical, civil, aerospace and structural engineers and designers. Some applications of these systems are pressure vessel valve, reinforced circular plates by radial and circumference supporter, composite plates, cylinder head cover, bulkhead plates in submarines, separated plates in aircraft, optical lenses, and acoustic transducers in rockets. For first time Woinosky[1], studied the problem of elastic stability of orthotropic circular plates. He introduced numeric results by using Bessel function for buckling of plates. Menk *et al*.[2], studied on variation of thickness on buckling of orthotropic rectangular plate. Laura *et al*. [3], found critical load of buckling for isotropic annular plate with constant thickness by optimized Ritz method. Imposed boundary condition of plate for inner edge either outer edge was under different supports. Cianco [4] studied on buckling of circular and annular isotropic plate with variable thickness that used as a part of submarine. This plate was considered with free support on inner edge and clamped support and resistant of rotation for outer edge. The thickness of plate is exponential function of its radius. He analyzed the plate by optimized Ritz method. Bremec *et al.* [5] also introduced one optimized rate of variation of thickness for buckling of isotropic plates which both in inner edge and outer edge was under constant radial load and thickness was varied in radial direction. Buckling function was in linear fashion and solved by numeric method under simply and clamped supported. Coutierrez *et al.* [6] considered buckling and vibration of the isotropic plate with variable thickness on elastic support using Ritz method, and obtained acceptable results. Liang *et al.* [7] found natural frequencies of one orthotropic circular and annular plate with variable thickness using Ritz method and compared with the result of finite element method that were good in agreement between them.

In this paper, buckling of orthotropic circular and annular plates with linear variation of thickness under constant compressive radial loading is studied. The boundary condition of annular plates are F-C plates (free inner, and clamped outer edge) or clamped circular solid ones, F-S plates (free inner, simply outer edge) or simply supported solid circular ones, and annular plates with free inner support and resistant elastic against rotation outer edge. Circular plates contain plates with clamped, simply and elastic resistant against rotation boundary condition. Solving buckling differential equation of orthotropic circular or annular plate with variable thickness is impossible by analytical method and it should be solved using numerical or energy method. For this reason optimized - Ritz method is used. The results of this method is more precisely than Ritz method. For optimization of Ritz method, one exponential parameter in approximate function is considered eigenvalues (buckling load factor) that is obtained, are minimized according to this exponential parameter. Furthermore, the comparison between results of this method and the results of finite element procedure is done. The effects of thickness variation, boundary conditions, young module ratio in radius and circumference axis, variation of ratio of inner radius to outer one on buckling load factor are considered.

2 Theory

2.1 Basic formulation of the problem

The formulation of the problem is derived under the following assumptions:

1. The plate is in the state of plane stress.
2. The stress-strain relationship follows of orthotropic material.

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3. The plate is thin, therefore the Kirchhoff assumptions incorporated.
4. The thickness is varied in the direction of radius of the plate.

Consider a circular annular plate with variable thickness h(r), a, b inner and outer radius, respectively as shown in Figure (1). For buckling analysis, the in-plane displacement u and v may be neglected and only out-of-plane deformation w is considered.

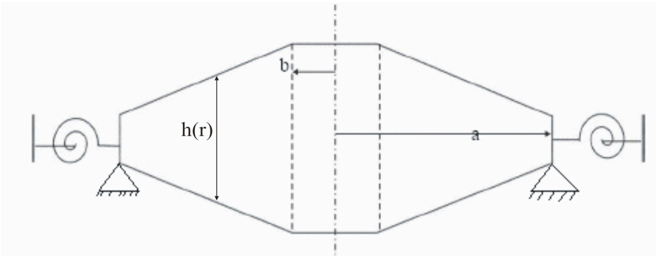

Fig. 1. Schematic view of annular plate with variable thickness

The governing energy functional can be given by:

$$J=U+V \tag{1}$$

Where U is the stored strain energy per unit volume that in the polar coordinate system for plane stress is given as follows [8]:

$$U=\frac{1}{2}\iiint_v \left(\sigma_r\varepsilon_r+\sigma_\theta\varepsilon_\theta+\tau_{r\theta}\gamma_{r\theta}\right) r\,dr\,d\theta\,dz \tag{2}$$

The work done by in-plane radial force is given as:

$$V=\frac{1}{2}\iint_A\left[N_r\left(\frac{\partial w}{\partial r}\right)^2+N_\theta\left(\frac{\partial w}{\partial \theta}\right)^2\right] r\,dr\,d\theta \tag{3}$$

In present study, the axial radial symmetry is assumed, so plate is independent of azimuthal variable:

$$V=\pi\int_b^a N_r\left(\frac{\partial w}{\partial r}\right)^2 r\,dr \tag{4}$$

For orthotropic annular plate, V is derived as follows [9]:

$$V=\pi\int_b^a \frac{-N_0}{h_a}\frac{a^{\beta+1}}{a^{2\beta}-b^{2\beta}}\left[r^{\beta-1}-\frac{b^{2\beta}}{r^{\beta+1}}\right]\left(\frac{\partial w}{\partial r}\right)^2 r\,dr \tag{5}$$

Also, using the stress-strain relation for orthotropic material, substituting into equation (1), one may obtain:

$$U=\frac{1}{2}\iiint_v \frac{z^2}{1-\upsilon_r\upsilon_\theta}\left[E_r\left(\frac{d^2w}{dr^2}\right)^2+2E_r\upsilon_\theta\left(\frac{d^2w}{dr^2}\right)\left(\frac{dw}{dr}\right)+E_\theta\left(\frac{dw}{rdr}\right)^2\right] r\,dr\,d\theta\,dz \tag{6}$$

where,

$$D_r(r)=\frac{E_r h^3(r)}{12(1-\upsilon_r\upsilon_\theta)}\quad,\quad \frac{E_\theta}{E_r}=\frac{D_\theta(r)}{D_r(r)}=\beta^2 \tag{7}$$

In above equation, $D_r(r), D_\theta(r)$ denote circumferential and radial bending stiffness of the plate, respectively.

In this study, annular and solid circular plates with continuously varying thickness are considered. The variation of thickness along the radius direction can be expressed as follows:

$$h(r)=h_o(1+\gamma(\frac{r}{a})^n) \tag{9}$$

where *ho* and *a* represent the thickness in the centre (inner radius for annular plate) and the outer radius of plate, respectively.

In order to consider the influence of non uniform thickness on buckling load factor, two values are assigned for the parameter n, n=0, 1 for uniform and linearly varying thickness, respectively. γ is non-dimensional geometric parameter that may be positive (centrally thinner circular plate) or negative (centrally thicker circular plate) and is defined as follows:

For circular solid plates:

$$\gamma=\frac{h_a}{h_o}-1 \tag{10}$$

For circular annular plates:

$$\gamma=\frac{h_b-h_o}{h_o(r_b)^n} \tag{11}$$

In above equation, r_b is the ratio of inner radius to the outer one ($=\frac{b}{a}$).

In the sake of convenience, the variables in equations (7) and (15) are transformed to dimensionless one, so the total potential energy functional is as follows:

$$\frac{a^2}{\pi D_0}J(w)=\int_{r_b}^1\left\{g(R)\left[\left(\frac{d^2w}{dR^2}\right)^2+2\upsilon_\theta\left(\frac{d^2w}{dR^2}\right)\left(\frac{dw}{RdR}\right)+\beta^2\left(\frac{dw}{RdR}\right)^2\right]-\frac{\lambda}{1-r_b^{2\beta}}\left[R^{\beta-1}-\frac{r_b^{2\beta}}{R^{\beta+1}}\right]\left(\frac{dw}{dR}\right)^2\right\}RdR \tag{12}$$

that,

$$g(R)=(1+\gamma R)^3,\quad D_0=\frac{E_r h_o^3}{12(1-\upsilon_r\upsilon_\theta)},\quad R=\frac{r}{a} \tag{13}$$

λ is the buckling load factor that is related to buckling load as follows:

$$\lambda=\frac{N_0 a^2}{D_0} \tag{14}$$

2.2 Optimized Ritz method

Regarding to this fact that Ritz method is an upper bound method, so determined eigenvalue is more than real one, therefore if one can optimized it somehow, the results will be closer to real one. In general, if the function which introducing the unknown quantity, is a linear combination of shape modes φ_n, as follows:

$$f(x)=\sum_{n=1}^N c_n\varphi_n(x) \tag{15}$$

where c_n is the unknown constants. Regarding to the idea of optimization, by performing the optimized Ritz method, it is quite convenient to approximate the displacement amplitude W(R) by means of a summation [4]:

$$f(x)=\sum_{n=1}^N c_n\varphi_n(x,k) \tag{16}$$

Note that *k* in above equation is the exponential optimization parameter. Regarding to the equation (15), out-plane displacement is given as follows:

$$w(R)=\sum_{i=1}^N c_i w_i(R,k) \tag{17}$$

By minimizing potential energy functional in Ritz method:

$$\frac{\partial J}{\partial c_i} = 0 \qquad i = 1,\ldots,N \tag{18}$$

Total number of N linear homogenous algebraic equations are generated that the unknowns are constants c_n. It forms the eigenvalue problem that the eigenvlaues are the values of buckling load parameter. The non-trivially condition leads to a transcendental equation in whose lowest root is the desired buckling load factor [4].

Since $\frac{\partial \lambda}{\partial k} = 0$, by requiring one is able to optimize the fundamental eigenvalue.

2.3 Buckling of annular circular plate

Regarding to the equation (15), $w_i(R)$ is defined as follows [9]:

$$w_i(R,k) = \left(a_i R^k + b_i R^{1+\beta} + 1\right)R^{i-1} \tag{19}$$

Unknown constants a_i and b_i are determined by applying boundary conditions.

2.3.1 Clamped outer edge (F-C) plate

For a clamped outer edge; in r=a or R=1, the governing boundary conditions are:

$$\begin{cases} w_i(1) = 0 \\ \dfrac{dw_i}{dR}(1) = 0 \end{cases} \tag{20}$$

By substitution eq. (19) into eq. (20), the a_i and b_i values are determined as follows:

$$a_i = \frac{-1-\beta}{1+\beta-k} \qquad b_i = \frac{k}{1+\beta-k} \tag{21}$$

2.3.2 Simply supported outer edge(S-F) plate

For a simply supported outer edge, the out-plane displacement w, must be satisfied the following conditions:

$$\begin{cases} w(1) = 0 \\ \dfrac{d^2 w}{dR^2}(1) + \upsilon_\theta \dfrac{dw}{dR}(1) = 0 \end{cases} \tag{22}$$

Regarding to the boundary condition in outer edge, the a_i and b_i values are determined:

$$\begin{aligned} a_i &= \frac{(1+\beta)(-2+2i+\beta+\upsilon_\theta)}{(-1-\beta+k)(-2+2i+k+\beta+\upsilon_\theta)} \\ b_i &= -\frac{k(-3+2i+k+\upsilon_\theta)}{(-1-\beta+k)(-2+2i+k+\beta+\upsilon_\theta)} \end{aligned} \tag{23}$$

2.3.3 Elastically restrained rotation

For the case of elastically restrained rotation:

$$\begin{cases} w_i(1) = 0 \\ \varphi \dfrac{dw_i}{dR}(1) = -g(R)\left[\dfrac{d^2 w_i}{dR^2}(1) + \upsilon_\theta \dfrac{dw_i}{dR}(1)\right] \end{cases} \tag{24}$$

the a_i and b_i values are determined as follows:

$$a_i = -\frac{Q_i - L_i}{S_i - L_i}, \quad b_i = -\frac{S_i - Q_i}{S_i - L_i} \tag{25}$$

In above equation, S_i, Q_i, L_i are defined as follows:

$$\begin{aligned} S_i &= (k+i-1)\left[1 + g(R)\varphi(-2+i+k+\upsilon_\theta)\right] \\ L_i &= (i+\beta)\left[1 + g(R)\varphi(-1+i+\beta+\upsilon_\theta)\right] \\ Q_i &= (i-1)\left[1 + g(R)\varphi(-2+i+\upsilon_\theta)\right] \end{aligned} \tag{26}$$

$\varphi = \dfrac{ak_\varphi}{D_0}$ represents the dimensionless flexibility coefficient. Using the equation (18), total number of N linear homogenous algebraic equations is generated. The non-trivially condition leads to set zero the determinant of coefficients matrix, as follows:

$$|A - \lambda B| = 0 \tag{27}$$

Where A_{ij} and B_{ij} are determined as follows:

$$A_{ij} = \int_{r_b}^{1} g(r)\left[\left(\frac{d^2 w_i}{dR^2}\right)\left(\frac{d^2 w_j}{dR^2}\right) + 2\upsilon_\theta\left(\frac{d^2 w_i}{dR^2}\right)\left(\frac{dw_j}{RdR}\right) + \beta^2\left(\frac{dw_i}{RdR}\right)\left(\frac{dw_j}{RdR}\right)\right]RdR + \varphi\left(\frac{dw_i}{dR}\right)\left(\frac{dw_j}{dR}\right) \tag{28}$$

$$B_{ij} = \frac{\lambda}{1-r_b^{2\beta}}\left[R^{\beta-1} - \frac{r_b^{2\beta}}{R^{\beta+1}}\right]\int_{r_b}^{1}\left(\frac{dw_i}{dR}\right)\left(\frac{dw_j}{dR}\right)RdR \tag{29}$$

In the above equation, for the case of simply supported and clamped in outer edge, φ=0 , φ=∞ respectively.

2.4 Buckling of circular solid plate

Regarding to equation (17), for circular solid plate $w_i(R)$ is defined as follows [9]:

$$w_i(R,k) = \left(a_i R^k + b_i R^{1+\beta} + 1\right)R^{2(i-1)} \tag{30}$$

In order to obtain a_i and b_i as constants, the following boundary conditions must be satisfied:

2.4.1 Clamped outer edge, (C-F) plate

$$b_i = \frac{k}{1+\beta-k},\quad a_i = \frac{-1-\beta}{1+\beta-k} \tag{31}$$

2.4.2 Simply supported outer edge, (S-F) plate

$$\begin{aligned} a_i &= \frac{(1+\beta)(-4+4i+\beta+\upsilon_\theta)}{(-1-\beta+k)(-4+4i+k+\beta+\upsilon_\theta)} \\ b_i &= -\frac{k(-5+4i+k+\upsilon_\theta)}{(-1-\beta+k)(-4+4i+k+\beta+\upsilon_\theta)} \end{aligned} \tag{32}$$

2.4.3 Elastically restrained rotation

$$a_i = -\frac{Q_i-L_i}{S_i-L_i} \qquad b_i = -\frac{S_i-Q_i}{S_i-L_i} \tag{33}$$

where,

$$\begin{aligned} S_i &= (k+2i-2)\left[1+g(R)K_\phi(-3+2i+k+\upsilon_\theta)\right] \\ L_i &= (2i+\beta-1)\left[1+g(R)K_\phi(-2+2i+\beta+\upsilon_\theta)\right] \\ Q_i &= 2(i-1)\left[1+g(R)K_\phi(-3+2i+\upsilon_\theta)\right] \end{aligned} \tag{33}$$

A $_{ij}$ and B_{ij} are determined as follows:

$$\begin{aligned} A_{ij} = \int_0^1 g(R)\Bigg[&\left(\frac{d^2w_i}{dR^2}\right)\left(\frac{d^2w_j}{dR^2}\right)+2\upsilon_\theta\left(\frac{d^2w_i}{dR^2}\right)\left(\frac{dw_j}{RdR}\right) \\ &+\beta^2\left(\frac{dw_i}{RdR}\right)\left(\frac{dw_j}{RdR}\right)\Bigg]+\varphi\left(\frac{dw_i(1)}{RdR}\right)\left(\frac{dw_j(1)}{RdR}\right) \end{aligned} \tag{34}$$

$$B_{ij} = \lambda R^{\beta-1}\int_0^1\left(\frac{dw_i}{dR}\right)\left(\frac{dw_j}{dR}\right)RdR \tag{35}$$

Like before, in the above equation, for the case of simply supported φ=0 and clamped in outer edge, φ=∞.

3. Numerical results

This section presents a number of numerical examples that shows the good performance of the proposed method, which was implemented in *Mathematica 5.1* computer program. The results of the developed optimized Ritz method are compared with some other results obtained from FE method. All calculation has been performed for E_r =10000 Mpa, υ_θ =.3, a=1 m, h_o=.08 m. It was revealed that convergent buckling load factor is obtained with 4-term series. Table (1) shows this convergence for centrally thicker annular orthotropic plate. In presenting results, the dimensionless buckling load factor is used. Value of this factor is obtained for plates of uniform and linearly continuously thickness.

Table (1): Convergence study for annular orthotropic plate

γ	r_b	m				
		1	2	3	4	5
-0.3	0.1	8.06227	7.82111	7.80585	7.65965	7.65696
	0.3	7.64248	7.26206	7.24417	7.15621	7.15612
	0.5	10.7017	10.6339	10.6283	10.6069	10.6066

Annular plate

Table (2) depicts the influence of parameters γ, β^2, r_b on the buckling load factor. It is observed with increasing the amount of β, buckling load factor is increased too. One observes that orthotropic plate ($\beta^2 > 1$) has more stiffness against buckling occasion in comparison to isotropic one. Furthermore increasing parameter γ toward positive values makes the buckling load factor to increase. Regarding to table (1), some values of r_b (r_b>.1) decreases the buckling load factor λ , meanwhile some other values makes it decrease. As shown in table (3), as it was expected, clamped boundary condition represents the highest value of factor while the simply supported one, shows the lowest. The influence of parameters γ, β^2, r_b is the same as clamped case.

Table 2. Buckling load factor variation with simply supported outer edge

β^2	γ	r_b				
		0.1	0.2	0.3	0.4	0.5
1	-0.3	7.65965	7.08443	7.15621	8.17407	10.6069
	-0.1	11.6538	11.1526	11.9511	14.4785	19.7288
	0	14.0102	13.6272	14.9707	18.5588	25.7602
	0.1	16.605	16.4169	18.4444	23.3258	32.8914
	0.3	23.5623	23.0035	26.8698	35.1226	50.8109
2	-0.3	12.2977	11.7989	11.3131	11.6165	13.4416
	-0.1	19.4974	18.8195	18.6013	20.0802	24.5068
	0	23.8244	23.0811	23.1413	25.5038	31.7732
	0.1	28.6657	27.8789	28.3333	31.807	40.3339
	0.3	39.9593	39.1814	40.8333	47.3052	61.7521
5	-0.3	23.331	23.2335	22.7974	22.1416	22.4007
	-0.1	39.3318	39.151	38.4959	38.0138	39.9237
	0	49.2861	49.0655	48.3052	48.0918	51.309
	0.1	60.6435	60.3648	59.3648	59.7502	64.6481
	0.3	87.8954	87.4584	86.6497	88.2489	97.7947
10	-0.3	39.4051	39.2595	39.2595	38.7452	37.9053
	-0.1	69.3379	69.0895	69.0894	68.2961	67.7093
	0	88.3379	88.4204	88.1414	87.2232	87.0211
	0.1	110.615	110.568	110.221	109.203	109.61
	0.3	164.852	164.774	164.275	163.176	165.608

Table 3. Buckling load factor variation with clamped outer edge

β^2	γ	r_b				
		0.1	0.2	0.3	0.4	0.5
1	-0.3	2.38864	2.08242	1.74291	1.45192	1.22152
	-0.1	3.40413	2.99726	2.59067	2.25996	2.00307
	0	3.99418	3.53995	3.10666	2.76324	2.50023
	0.1	4.64426	4.14536	3.69047	3.33985	3.007638
	0.3	6.13934	5.56135	5.8046	4.7343	4.48982
2	-0.3	3.865	3.70686	3.37779	2.95803	2.54578
	-0.1	5.91459	5.67527	5.2282	4.70576	4.21785
	0	7.14892	6.86634	6.36338	5.79828	5.28355
	0.1	8.53568	8.20873	7.65316	7.05245	6.51991
	0.3	11.8043	11.3873	10.7398	10.0932	9.5504
5	-0.3	7.44953	7.42653	7.29997	6.95221	6.35169
	-0.1	12.4039	12.3628	12.1571	11.6415	10.8251
	0	15.5186	15.467	15.2169	14.6124	13.6953
	0.1	19.1024	19.0392	18.7421	18.0519	17.0371
	0.3	27.8088	27.7192	27.3225	26.4616	25.2822
10	-0.3	12.7613	12.7592	12.7332	12.5904	12.1299
	-0.1	22.434	22.4299	22.3826	22.1401	21.4237
	0	28.6802	28.6739	28.6134	28.3129	27.4558
	0.1	35.9679	35.9621	35.889	35.5252	34.5213
	0.3	54.0108	54.0021	53.8957	53.3922	52.0821

Tables (4-6) depict the influence of rotational constant φ on buckling load factor for different value of β^2. It is revealed that when $\varphi \to 0$, the boundary condition is closer to simply supported case and the loading factor shows the lower than when $\varphi \to \infty$ the outer edge is clamped and stiff against rotation.

Table 4. Buckling load factor variation with edge elastically restrained $(\beta = 1)$

γ	φ	r_b				
		0.1	0.2	0.3	0.4	0.5
-0.3	0	2.39267	2.08653	1.74474	1.45248	1.22165
	10	7.02073	6.47188	6.4328	7.14899	8.92773
	∞	7.78485	7.17253	7.20876	8.21381	10.6283
-0.1	0	3.4185	3.00976	2.59598	2.26171	2.00357
	10	10.1649	9.62109	9.92857	11.3968	14.4472
	∞	11.7928	11.2945	12.0246	14.5139	19.7524
0	0	4.0144	3.55674	3.11384	2.76572	2.50097
	10	11.7538	11.2724	11.8074	13.6684	17.3273
	∞	14.1309	13.7553	15.0361	18.5922	25.7854
0.1	0	4.66952	4.16614	3.6995	3.34309	3.07739
	10	13.3724	12.9663	13.7373	15.9742	20.1736
	∞	16.7082	16.5284	18.5018	23.357	32.9186
0.3	0	6.17125	5.5881	5.0932	4.73918	4.49148
	10	16.6434	16.4146	17.6377	20.5221	25.5586
	∞	22.6	23.0801	26.9097	35.1488	50.8404

Table 5. load factor variation with edge elastically restrained $(\beta = \sqrt{2})$

γ	φ	r_b				
		0.1	0.2	0.3	0.4	0.5
-0.3	0	3.86494	3.70778	3.37948	2.95904	2.54614
	10	11.1494	10.7466	10.2355	10.2921	11.4907
	∞	12.3941	11.9712	11.429	11.7022	13.4887
-0.1	0	5.92078	5.68816	5.23808	4.71045	4.21952
	10	16.3925	15.9406	15.5214	16.0831	18.3679
	∞	19.6007	19.0235	18.7609	20.168	24.5558
0	0	7.16249	6.88791	6.37831	5.80537	5.28613
	10	19.173	18.6768	18.3597	19.2084	22.0132
	∞	23.9331	23.2797	23.2917	25.5881	31.8244
0.1	0	8.55772	8.23958	7.67409	7.06221	6.52353
	10	22.0216	21.1903	21.2775	22.4076	25.679
	∞	28.762	28.062	28.4736	31.8876	40.388
0.3	0	11.8246	11.4362	10.7724	10.1088	9.56313
	10	27.8305	27.2711	27.2789	28.8982	32.897
	∞	40.0193	39.3203	40.9454	47.3759	61.8092

Table 6. load factor variation with edge elastically restrained $(\beta = \sqrt{5})$

γ	φ	r_b				
		0.1	0.2	0.3	0.4	0.5
-0.3	0	7.44981	7.42763	7.30188	6.95214	6.35201
	10	20.398	20.3231	19.9702	19.3557	19.2569
	∞	23.5367	23.4633	23.0736	23.396	22.5735
-0.1	0	12.4127	12.3628	12.1602	11.6479	10.83
	10	31.3392	31.2815	30.8997	30.2359	30.5775
	∞	39.4941	39.3991	38.8456	38.3133	40.103
0	0	15.5204	15.469	15.2275	14.6276	13.70421
	10	37.136	37.0678	36.654	36.058	36.6338
	∞	49.4286	49.3058	48.672	48.401	51.4971
0.1	0	19.1036	19.0485	18.764	18.0745	17.0509
	10	43.1385	43.0614	42.6181	42.0768	42.8725
	∞	60.7884	60.6234	59.9327	60.0692	64.8462
0.3	0	27.8251	27.7559	27.3754	26.5071	25.3083
	10	55.7733	55.6721	55.182	54.7283	55.8625
	∞	88.0433	87.7881	87.0796	88.5781	98.01

3.2 circular solid plate

Figures (2) and (3) illustrate the variation of bucking load factor respect to boundary condition case, orthotropic and geometry β^2 and γ.

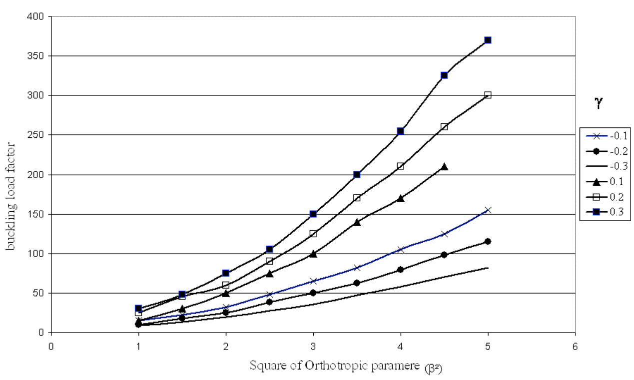

Fig. 2 variation of buckling load factor respect to orthotropic parameter and non-dimensional geometric parameter with clamped outer edge

As shown, plate with $\gamma < 0$ (centrally thicker solid plate, $\frac{ha}{h0} < 1$) has lower buckling load factor than plate with $\gamma > 0$ (centrally thinner circular plate, $\frac{ha}{h0} > 1$). Buckling factor pattern for all boundary conditions are similar.

Figure (4) illustrates the influence of rotational constant φ on buckling load factor for different value of β^2. For different boundary conditions, the obtained result is the same as the annular plate In fact, for simply supported outer edg, the evaluated buckling load factor is lower than clamped and elastically restrained against rotation in outer edge.

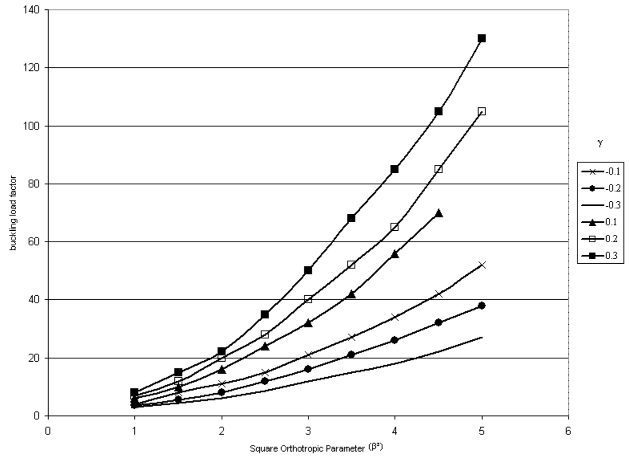

Fig. 3. Variation of buckling load factor respect to orthotropic parameter and non-dimensional geometric parameter with simply supported outer edge

As shown in Table (6), the results obtained from present method, compare very well with obtained from finite element procedure. This confirms the accuracy of Optimized Ritz method in buckling analyzing of circular plates.

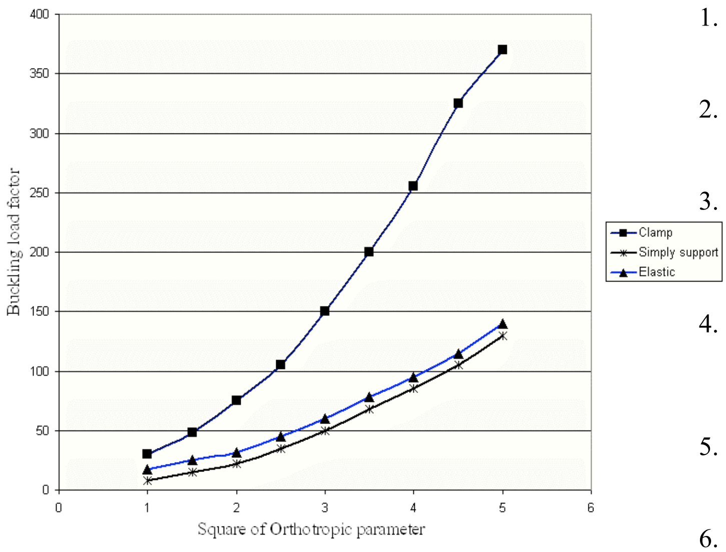

Fig. 4. Variation of buckling load factor respect to orthotropic parameter for different boundary conditions

Table 7. Comparison of results for buckling load factor in Optimized Ritz method (I) and FEM (II)

β^2		γ -0.3	-0.2	-0.1	0	0.1	0.2	0.3
1	I	8.05	10.03	12.24	14.68	17.36	20.28	23.43
	II	7.8	9.825	12.11	14.68	17.53	20.69	24.15
1.4	I	9.87	12.43	15.32	18.54	22.1	26.04	30.26
	II	9.72	12.31	15.24	18.54	22.21	26.28	30.75
1.8	I	11.56	14.69	18.25	22.24	26.67	31.57	36.94
	II	11.55	14.67	18.23	22.23	26.7	31.66	37.11

4 Conclusion

The buckling analysis of orthotropic circular annular and solid plates under uniform radial compression loading with uniform and linearly varying thickness was presented. The inner edge is free while the outer edge is under different types of classical boundary conditions and also with edges elastically restrained against rotation. This is implemented by optimized Ritz method. In this method, an optimization exponential parameter is utilized that buckling load factor was minimized respect to it. Following are some of the concluding remarks:

- Increasing orthotropic parameter and radius ratio (inner to outer radius), increases the resistance of plate against buckling phenomena.
- Plate with clamped boundary condition exhibits the higher value of the buckling parameter, while the simply supported case shows lowest. Also plate with edges elastically restrained against rotation has value of between two boundary condition cases.
- Centrally thinner circular plate has higher buckling parameter than centrally thicker one.

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