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Optimization of warehousing and transportation costs, in a multi-product multi-level supply chain system, under a stochastic demand

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Abstract- The centralized management approach provides a general view to set a better coordination between the elements of the supply chain, and look for the equilibrium between the stock and the shipped quantities. This paper describes a stochastic model predictive control algorithm to optimize the warehousing cost and the shipping cost on the same time. We consider a multi-level and multi-product supply chain dealing with a stochastic demand. We solve the stochastic model predictive control problem by a dynamic programming to determine the optimal policies to minimize the costs of shipping and warehousing products.

Key words: Supply Chain; Model Predictive Control; Stochastic demand; multi-product; multi-level

1 Introduction

The companies have been more then ever hunting waste and optimizing their costs through all the supply chain process. Different approaches are available, however the centralized approach in management of supply chain is sometimes more profitable then the decentralized one. The work of Perea-Lopez, [7] showed that profit could increases of up to 15 per cent by proposing a model predictive control strategy to optimize supply chain. The Model Predictive Control (MPC) is a methodology widely used typically in systems with slow dynamics as chemical process plant and supply chain (Qin and Badgwell, 1996). It is a solution to optimize closed loop performance under constraints and/or nonlinearity of multivariable dynamical systems (Fig. 1).

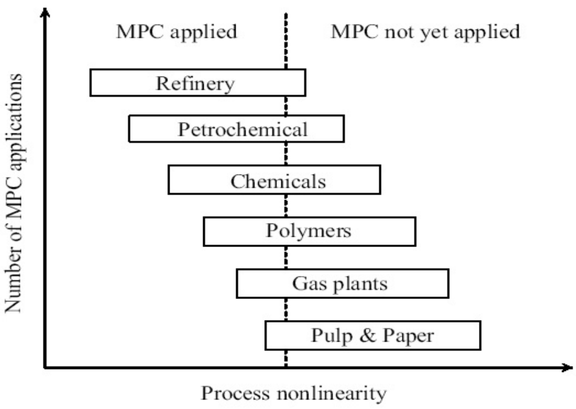

Fig. 1: Industrial applications of MPC

It is unique in providing computationally tractable optimal control laws by solving constrained receding horizon control problems online. As is frequently the case, the idea of MPC appears to have been proposed long before MPC came to the forefront (Propoi, 1963; Rafal and Stevens, 1968; Nour-Eldin, 1971). Not unlike many technical inventions, MPC was first implemented in industry , under various guises and names as showed in Fig. 2 , long before a thorough understanding of its theoretical properties was available. Academic interest in MPC started growing in the mid eighties, particularly after two workshops organized by Shell (Prett and Morari, 1987; Prett et al., 1990).

Company	Product name	Description
Aspen Tech	DMC	Dynamic Matrix Control
Adersa	IDCOM	Identification and Command
	HIECON	Hierarchical Constraint Control
	PFC	Predictive Functional Control
Honeywell Profimatics	RMPCT	Robust Model Predictive Control Technology
	PCT	Predictive Control Technology
Setpoint Inc.	SMCA	Setpoint Multivariable Control Architecture
	IDCOM-M	Multivariable
Treiber Controls	OPC	Optimum Predictive Control
Shell Global	SMOC-II	Shell Multivariable Optimizing Control
ABB	3dMPC	
Pavillion Technologies Inc.	PP	Process Perfecter
Simulation Sciences	Connoisseur	Control and Identification Package

Fig. 2: Industrial technology implementing MPC

The understanding of MPC properties generated by pivotal academic investigations (Morari and Garcia, 1982; Rawlings and Muske, 1993) has now built a strong conceptual and practical framework for both practitioners and theoreticians. [8]. The MPC is usually used in supply chain management, under constraints like buffer limits and shipping capacities limits, based on approximations which make the future values of disturbance exactly

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as predicted, thus no recourse is available in the future. However, most real life applications are not only subject to constraints but also involve multiplicative and/or additive stochastic uncertainty. Earlier work tended to ignore information on the distribution of model uncertainty, and as a result addressed control problems sub-optimally using robust MPC strategies that employ only information on bounds on the uncertainty. Increasing demands for optimality in the presence of uncertainty motivate the development and application of MPC that takes explicit account of both omnipresent constraints and ubiquitous stochastic uncertainty [2] : which is the main idea of the Stochastic Model Predictive Control. In this paper, we study the application of the Stochastic Model Predictive Control on a multi-product multi-level supply chain to determine the optimal control policies by dynamic programming.

2 Problem description

The general structure of the supply chain that we analyse, consists of four different levels showed in Fig. 3. The first level is retailers, who take the customer demands and try to satisfy it according to their level stock. The second level is the distribution centers (DC),they distribute the products to the retailers to fulfil their needs. The Plant Warehouses are the third level in the supply chain, their stock capacity is bigger then the distribution centers, and the fourth level is the plants. We consider a multi-product supply chain, and to simplify the problem we assume that the products are independents from each other. Each node in the supply chain has to manage its storage level and at the same time to satisfy the demand of its corresponding nodes from the former level.

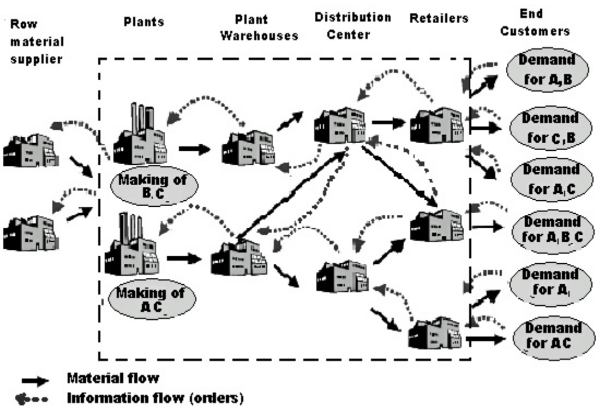

Fig. 3: Structure of supply chain network

The problem consists on finding the equilibrium between the quantities of the product transported from each node and the one stored to fulfil a stochastic demand, which lead to optimize the shipping cost and the warehousing cost simultaneously. This problem can be stated as follows: given a multi-level and multi-product supply chain, and for a stochastic demand from customers over a finite time horizon, the goal is to define the optimal costs of the warehousing and the shipping. In this paper we develop a stochastic model predictive control to describe this problem, and we propose the dynamic programming to determine the optimal control policies to minimize the storage and the shipping costs.

3 State function

In a general stochastic model predictive control, the linear dynamical system over a finite time horizon is like follows:

$$x_{t+1} = Ax_t + Bu_t + d_t \qquad t = 0, \dots, T-1 \qquad (1)$$

with :
x_{t+1}: is the state of the system at time t.
u_t: is the input (the control) at time t.
d_t: is the process noise (or exogenous input)at time t.
The process noise in this problem is the stochastic demand, then d_t will express the demand. The control u_t is representing the quantity transported of the products, and the x_t represents the amount of the product present at time t

To present the different terms of the equation Eq. 1 adapted to this problem, we propose an example of a three-products (P1, P2, P3) supply chain, like shows the Fig. 4 . We present the four levels of the supply chain by nodes: the node n_1 to represent the plant, and the nodes n_2 and n_3 to represent the warehouses, the nodes n_4, n_5 and n_6 for the distribution centers, and the rest of the nodes to represent the retailers. The links u_i present the possibility and the sense of the shipping between the nodes.

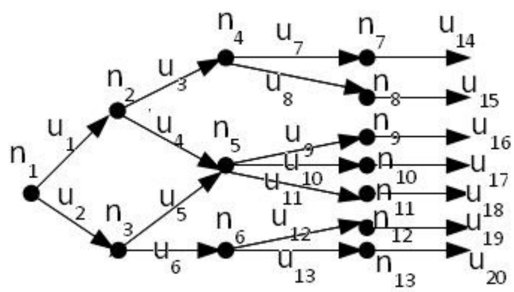

Fig. 4: Example of supply chain

The state variable x is a matrix:

$$x_t \in R_{n*p} \qquad (2)$$

with:
n: is the number of nodes in the supply chain,
p: is the number of category product.

In the example of the Fig.4, we have $n = 11$ and we supposed that we have three products (P_1, P_2, P_3) $p = 3$. Then the matrix $x_t \in R_{11*3}$,where $x_{i,j}$ presents the amount of the product j presents at the node i.

$$x = \begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,1} & x_{3,2} & x_{3,3} \\ x_{4,1} & x_{4,2} & x_{4,3} \\ x_{5,1} & x_{5,2} & x_{5,3} \\ x_{6,1} & x_{6,2} & x_{6,3} \\ x_{7,1} & x_{7,2} & x_{7,3} \\ x_{8,1} & x_{8,2} & x_{8,3} \\ x_{9,1} & x_{9,2} & x_{9,3} \\ x_{10,1} & x_{10,2} & x_{10,3} \\ x_{11,1} & x_{11,2} & x_{11,3} \end{pmatrix}$$

Then the column j describes the amount of the product P_j for all the nodes in the supply chain. In the same way, the line i in the matrix x describes the quantity in stock for each product presents at the node i.
The next variable to define in the equation Eq.1 is the control u_t. It is a matrix describing the amount of product shipped.

$$u_t \in R_{m*p} \tag{3}$$

with:
m: is the number of links in the supply chain,
p: is the number of category product.
In the example of the Fig.4, we have $m = 20$ and we have $p = 3$. Then the matrix $u_t \in R_{20*3}$,where $u_{i,j}$ presents the amount of the product j shipped by the link i. If a product j can't be shipped by a link i, then we will add to the conditions of the problem : $u_{i,j} = 0$

For the matrix presenting the demand on each node we have :

$$d_t \in R_{n*p} \tag{4}$$

with:
n: is the number of nodes in the supply chain,
p: is the number of category product.
Finally, the matrix B express the incoming and outgoing node incidence.

$$B \in R_{n*m} \tag{5}$$

with:
n: is the number of nodes in the supply chain,
m: is the number of links in the supply chain. We split the matrix B on two matrix B^{in} and B^{out} expressed like follows:

$$B_{ij}^{in(out)} = \begin{cases} 1 & \text{; link } j \text{ enters (exits) node } i \\ 0 & \text{; otherwise} \end{cases} \tag{6}$$

For the example in Fig.4 the link u_4 enters the node n_5 then $B_{5,4}^{in} = 1$ and the link u_{18} enters the node n_{11} then $B_{11,18}^{out} = 1$

Then the expression of the dynamic of the system, will be like presented in Eq.7 :

$$x_{t+1} = x_t + B^{in}.u_t - B^{out}.u_t + d_t \quad t = 0, \ldots, T-1 \tag{7}$$

4 Stochastic control problem

4.1 Objective function

The objective funtion of this problem, is the cost of storage and shipping, which will be expressed like showed in the Eq. 8

$$J = E(\sum_{t=0}^{T-1} l_t(x_t, u_t) + l_T(x_T)) \tag{8}$$

We explicit the stage cost functions like:

$$l_t(x_t, u_t) = S(u(t)) + W(x(t)) \tag{9}$$

With:
$S(u(t))$: Transportation cost depends of $u(t)$ the amount of commodity transported
$W(x(t))$: Storage cost depends on the $x(t)$ amount of commodity at warehouses

We consider a linear form of the cost functions :

$$S(u(t)) = Tr(R_t u_t) \quad \text{; with } R_t \in R_{p*m} \tag{10}$$

And for the warehousing cost :

$$W(x(t)) = Tr(Q_t x_t) \quad \text{; with } Q_t \in R_{p*n} \tag{11}$$

4.2 Control policy

Let be X_t the state history up to time t,

$$X_t = (x_0, ..., x_t) \tag{12}$$

Then the expression of the causal state-feedback control:

$$u_t = \Phi_t(X_t) = \psi_t(x_0, d_0, \ldots, d_{t-1}) \quad t = 0, \ldots, T-1 \tag{13}$$

The stochastic control problem, is to choose control policies $\Phi_0, \ldots, \Phi_{T-1}$ to minimize J, subject to constraints.

$$min(J) = min(E(\sum_{t=0}^{T-1} l_t(x_t, u_t) + l_T(x_T))) \tag{14}$$

Regarding the constraints:
The buffer limits: (could allow $x_i(t) < 0$, to represent back-order)

$$0 \leq x_i(t) \leq x_{max} \tag{15}$$

The link capacities:

$$0 \leq u_i(t) \leq u_{max} \quad (16)$$

And as one can not ship out what is not on hand :

$$B^{out}u(t) \leq x(t) \quad (17)$$

5 Solution

We propose to resolve the stochastic model predictive control problem by dynamic programming. Let $V_t(X_t)$ be the optimal value of objective.

$$V_T(X_T) = l_T(x_T) \quad (18)$$

and ,

$$J^* = E(V_0(x_0)) \quad (19)$$

To simplify the resolution of the problem, we consider the problem as linear quadratic stochastic control for a two-level and one-product supply chain. And we assume: $U_t = R_m$
$x_0, d_0,\ldots,d_{T-1}$ are independent, with (to simplify the expressions)

$Ex_0 = 0,\ Ed_t = 0,\ Ex_0x_0^T = \Sigma,\ Ed_td_t^T = D_t$

Then, we express the cost functions in their convex quadratic form :

$$l_t(x_t, u_t) = x_t^T Q_t x_t + u_t^T R_t u_t \quad (20)$$

$$l_T(x_T) = x_T^T Q_T x_T \quad (21)$$

Let $V_t(x_t)$ be the optimal value of our objective (quadratic):

$$V_t(x_t) = x_t^T P_t x_t + q_t; \qquad t = 0, \ldots, T \quad (22)$$

Using Bellman recursion:

$$P_T = Q_T, q_T = 0; fort = T-1, \ldots, 0 \quad (23)$$

We get :

$$\begin{aligned} &V_t(z) = \inf_v z^T Q_t z + v^T R_t v + \\ &+\mathbf{E}((Az + Bv + d_t)^T P_{t+1}(Az + Bv + d_t) + q_{t+1}) \end{aligned} \quad (24)$$

And it works out to [1]

$$\begin{aligned} &P_t = A^T P_{t+1} A - \\ &-A^T P_{t+1} B (B^T P_{t+1} B + R_t)^{-1} B^T P_{t+1} A + Q_t \end{aligned} \quad (25)$$

and

$$q_t = q_{t+1} + \mathbf{Tr}(D_t P_{t+1}) \quad (26)$$

wich define all the variables to express $V_t(x_t)$
And for the optimal policy, we found it as a linear state feedback:

$$\Phi_t^* = (x_t) = K_t x_t \quad (27)$$

with :

$$K_t = -(B^T P_{t+1} B + R_t)^{-1} B^T P_{t+1} A \quad (28)$$

Finally, the expression of the optimal cost is :

$$\begin{aligned} J^* &= \mathbf{E}V_0(x_0) && (29)\\ &= \mathbf{Tr}(\Sigma P_0) + q_0 && (30)\\ &= \mathbf{Tr}(\Sigma P_0) + \Sigma_{t=0}^{T-1}\mathbf{Tr}(D_t P_{t+1}) && (31) \end{aligned}$$

Then in general for a multi-level and multi-product supply chain with a stochastic demand, V_t can be found by backward recursion for $t = T-1, \ldots 0$,

$$\begin{aligned} &V_t(X_t) = inf_{v\in U}(l_t(x_t, v) + \\ &+E(V_{t+1}((X_t, x_t + Bv + d_t))|X_t)) \end{aligned} \quad (32)$$

and by the same way we have the expression of the optimal policy :

$$\begin{aligned} &\Phi_t^*(X_t) = argmin_{v\in U}(l_t(x_t, v) + \\ &+E(V_{t+1}((X_t, x_t + Bv + d_t))|X_t)) \end{aligned} \quad (33)$$

6 Conclusions

In this paper, we proposed to optimize the shipping and the warehousing costs in a multi-product and multi-level supply chain under a stochastic demand. We presented how to optimize the shipping cost and in the same time the storage cost in a supply chain we need a centralized approach in supply chain management, which is allowed by the model predictive control. And to express the uncertainty in the demands, we used the stochastic model predictive control which lead us to determine the control policies to minimize the objective function J under the constraints of shipment and storage limits.

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