Multi-objective optimization on mix proportions of HSHPC applied to SRC composite structures

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Abstract – The fine bond behaviour between steel shape and concrete is the base of their cooperation. For designing high strength and high performance concrete (HSHPC) applied to steel reinforced concrete (SRC) structures, a mathematical model of multi-objective and nonlinear optimization is established based on the bond-slip theory between steel shape and concrete, which takes the materials’ cost and bond strength as objective functions, and takes the other performance indexes of HSHPC as constraint conditions. Optimal mix design is implemented with an application of convergent sequential quadratic programming and based on Matlab language. The optimized concrete mix proportions can satisfy all the specifications, and especially can raise the bond strength between steel shape and concrete and reduce unit cost of HSHPC.

Key words: SRC structure; HSHPC; mix proportion; optimization; multi-objective.

1 Introduction

Modern engineering structure advanced along the trend of long-span, heavy-load, high-rise. Attention to the durability of concrete structures promotes tremendous development in the technology of high strength and high performance concrete (HSHPC) \cite{1,2}. The design of concrete mix proportions is changing from traditional strength design to multi-performance design that includes strength, durability and working performance etc. Traditional design, which depends on a method of experiential semi-quantitative concrete mix proportions, will be renovated from feasibility design to optimization design. Steel reinforced high strength and high performance concrete structure is a kind of steel and concrete composite structure that applies HSHPC to steel reinforced concrete (SRC) structure. The main idea is using the good mechanical property and durable property of HSHPC to improve the cooperative performance between steel shape and concrete and increase security and durability of structures during service time \cite{3}.

This paper discusses multi-objective and nonlinear optimization design on mix proportions of HSHPC applied to SRC structures to improve synthetically performance of the structure. Design objective is transformed from traditional design of single strength target to multi-objective and nonlinear design considering working property, strength, the cooperative performance between steel shape and concrete, economic cost and endurance. For designing HSHPC applied to SRC structures, a multi-objective and nonlinear optimal mathematic model is established based on the bond-slip theory between steel shape and concrete, which takes the materials’ cost and ultimate bond strength as objective functions, and takes the other performance indexes of HSHPC as constraint conditions. The optimum problem is solved by convergent sequence quadratic programming. As a result, the choice of raw material parameters and the control of construction quality will be controlled by computer automatically.

2 Key performance indexes of HSHPC

2.1 Pore structure and interface transition layer

On mesoscopic level, concrete can be regarded as composition of coarse aggregate, hardened sanded cement grout and transition region between them. Certain of concrete material’s performance indexes such as strength, deformation ability, permeability and endurance are influenced by its pore structure and interface transition layer \cite{4}.

For capillary pore:
\[
\frac{W/C - \alpha(W/C^*)}{(W/C^*)} / [D_W \times V_c] = \frac{V_c}{D_W}. \tag{1}
\]

For gelled pore:
\[
\frac{mgV_g \times \alpha}{V_c + (W/C)/D_W}. \tag{2}
\]
where \( W/C^* \) is critical water cement ratio when cement is hydrated wholly; \( D_W \) is density of pore liquid in freshly mixed slurry; \( V_c \) is specific volume of unhydrated cement; \( V_g \) is average specific volume of gelation water; \( \alpha \) is percentage of hydrated cement; \( mg \) is weight of gelation water in a unit weight cement grout hydrated wholly.

According to the above model, volume of capillary pore in a unit weight cement grout is the difference between total volume and the sum volume of unhydrated cement and cement hydrate, which is denoted as \( [W/C - (W/C^*)\alpha]D_W \). Volume of gelled pore in a unit weight cement grout is \( mg \cdot V_g \cdot \alpha \).

According to the relation of water cement ratio and porosity mentioned above, it is clear that porosity of cement grout increases with increasing of water cement ratio.

Based on experiments the relation of compression strength and porosity of concrete is expressed as the following equation, which illustrates that concrete strength and Young’s modulus drop along with increase of porosity.

\[
f_c = a(1 - \rho)^3
\]

where \( f_c \) is compression strength of concrete; \( \rho \) is porosity; \( a \) is sanded cement grout strength when porosity equals to zero.

The relation of permeation ratio and pore structure can be expressed by an empirical equation as follows

\[
K = 1.684\alpha^{3.284} \times 10^{-22}
\]

where \( K \) denotes permeation ratio; \( r_{\infty} \) denotes maximum continuous radius, which is defined by the point of greatest gradient on mercury intrusion experiment curve.

The relation of permeation ratio, porosity and hydraulic radius is as follows

\[
\log K = 38.45 + 4.08 \log (\varepsilon r_n^2)
\]

where \( \varepsilon \) denotes porosity; \( r_n \) denotes hydraulic radius.

Mcha and Mamohan adopted four pore parameters to establish relation with permeation ratio

\[
K = \exp(3.84V_1 + 0.2V_2 + 0.56TD + 8.09MTP - 2.53)
\]

where \( V_1 \) is volume ratio of pore greater than 1320 Å and total pore; \( V_2 \) is volume ratio of pore between 290 Å to 1320 Å and total pore; \( TD \) is critical aperture, \( MTP \) is ratio of total volume and hydrating degree.

### 2.2 Simplified model and method to improve concrete strength

Concrete can be simplified as composition of hardened sanded cement grout and coarse aggregate, which link together through bonding effect. The simplified model is shown in Figure 1. When bearing external force, the two portions work together and have the same stress. Because Young’s modulus of them is not equal, the two portion’s strain is not the same, total strain is sum of the two portions, i.e.

\[
\begin{align*}
R_C &= R_A (1 - P) + R_S \frac{R_S}{E_S} P \\
R_C &= R_A = R_S.
\end{align*}
\]

Defining \( n = \frac{E_A}{E_S} \), then Young’s modulus of concrete is given by

\[
E_C = \frac{nE_S}{(n-1)p + 1}.
\]

Defining \( \beta = \frac{n}{(n-1)p+1} \), we can write

\[
E_C = \beta E_S
\]

where \( E_C, E_S, E_A \) denote Young’s modulus of concrete, coarse aggregate and hardened sanded cement grout respectively; \( P \) denotes grout’s volume content; \( n \) denotes Young’s modulus ratio of aggregate and grout; \( R_C, R_S, R_A \) denote concrete’s total stress, aggregate’s stress and grout’s stress respectively.

The above analysis indicates that cement grout and aggregate both affect concrete strength. Equations (7)–(9) and Figure 2 reflect the relationship of \( P \) and \( n \). In Figure 2, area \( \Delta_{\text{normB}} \) represents high strength concrete, in which cement grout’s strength is
higher than aggregate. The following are methods to improve concrete strength.

a. To improve P

In equations (7)–(9), n is definitive after materials to produce concrete are picked up. Within $\Delta_{n=0}t$ area shown in Figure 2, raising cement quantity can improve $P$. On the other hand, the improvement is not so effective. When $n$ equals to 0.5, $P$ increases to 0.6 from 0.3, accordingly $\beta$ increases to 0.701 from 0.588, and cement quantity is doubled but strength is raised only 17%, which means materials waste. This method is infeasible in actual engineering.

b. To improve n

Improving $n$ means improvement of aggregate strength. In like manner, when $P = 0.5$, $n$ increases to 0.4 from 0.2, accordingly $\alpha$ increases to 0.571 from 0.333, and improvement degree is 71.5%, which means applying high strength and high quality aggregate is the key point in producing high strength and high performance concrete.

c. To improve $E_S$

With regard to high strength and high performance concrete, coarse aggregate strength is lower than cement grout. Concrete damage lies on aggregate strength, when cement grout strength increases to a certain extent concrete strength will not be improved any more.

### 2.3 Bond strength between steel and concrete

In SRC structures there exists bonding action between steel shape and concrete, which are two kinds of materials with different performance. Bonding action enables stress transfer effectively each other on the interface of steel and concrete and establishes working stress needed to fulfill loading capacity of structures in steel and concrete [5]. Bond strength is a kind of shear force from macroscopic effect, the experiments showed that the distribution of bond stress on the interface of steel and concrete.

Based on the theory of mechanics of materials, the distribution of bond stress along steel surface is given as follows

$$\tau(x) = \frac{A_s}{u} \frac{d\sigma_x}{dx} = \frac{A_s}{u} E_s \frac{d\varepsilon_x}{dx} = -k_1 A_s \varepsilon_{max} \frac{E_s}{u} e^{-k_2 x}$$

(10)

where $E_s$ is elastic modulus of steel; $d\sigma_x/dx, d\varepsilon_x/dx$ are steel cross-section normal stress and normal strain increment along anchorage length respectively; $\sigma_x, A_s$ and $u$, for full section, are average stress, area and perimeter of steel section respectively, for flange or web, are average stress, area, perimeter of flange or web section respectively.

The distribution of slip between the interface of steel and concrete is given as follows

$$s(x) = s_{max} e^{-k_2 x}$$

(11)

where $s(x)$ is the slip quantity at different part of cross-section along anchorage length; $s_{max}$ is the maximum slip quantity along anchorage length; $k_2$ is characteristic exponent of slip distribution.

The bond-slip pull-out test of SRC members indicates that on the interface of steel and concrete the relationship of bond strength and concrete cube compression strength is as follows [6, 7]:

$$\tau_u = 0.0130 f_{cu} + 1.2150 C_a/h_a - 0.0640 l_a/h_a + 1.8682$$

(12)

where $\tau_u$ is ultimate bond strength; $f_{cu}$ is concrete cube compression strength; $C_a$ is thickness of covering layer; $h_a$ is sectional height; $l_a$ is the embedded depth of steel shape.

### 3 Multi-objective optimization design of concrete mix proportions

#### 3.1 Mathematical model

Mix proportion optimization design of HSHPC applied to SRC structures is a nonlinear multi-objective problem. In order to establish the relationship of every objective function and constraint conditions and finish optimum solution, multi-objective optimization technique is applied. The mathematical model of multi-objective optimization for concrete mix proportions is shown as follows [8]:

$$\begin{align*}
\min F(x), \\
\min G(x), \\
\text{s.t. } s_i(x) &\leq 0, \quad i = 1, 2, \ldots, m \\
h_i(x) &= 0, \quad j = 1, 2, \ldots, l(l < n) \\
x_1 &\leq x \leq x_u
\end{align*}$$

(13)

where $x = [x_1, x_2, \ldots, x_n]^T$ is decision-making vector including $n$ components; $F(x), G(x)$ is objective function vector; $s_i(x) \leq 0, h_j(x) = 0$ is constraint conditions. The decision-making Vector satisfying all restraint conditions is called feasible solution or feasible point, and the aggregate of all feasible points is called feasible aggregate. All optimization solution procedure is to find one point $x^*$ to make objective function obtain extremum in this point. $x^*$ is the optimal solution to the problem of concrete mix proportions accordingly.

#### 3.2 Constraint conditions

In this paper concrete raw material is composed of cement, sand, macadam, mineral admixture, high efficiency water reducing agent and water, which are calling six components of concrete. Mix proportion design is to determine the consumption of each component, the consumption of these six kinds of components are expressed as $x_1, x_2, \ldots, x_6$ individually [9].
a. The limits of each component consumption

\[ x_{il} \leq x_i \leq x_{iu}, \ i = 1, 2, \ldots, 6 \]  

(14)

where, \( x_{il} \), \( x_{iu} \) are lowest limit and maximal limit for \( x_i \) respectively.

b. The restriction of water gel ratio

\[ k_1 \leq x_6 / (x_1 + x_4) \leq k_u \]  

(15)

where, \( x_6 / (x_1 + x_4) \) is the ratio of water and gel material (the sum of cement and mineral admixture); \( k_1, k_u \) are lowest limit and maximal limit for water gel ratio respectively.

c. Sand ratio

\[ S_l \leq x_2 / (x_2 + x_3) \leq S_u \]  

(16)

where, \( x_2 / (x_2 + x_3) \) is the ratio of sand and coarse aggregate; \( S_l, S_u \) are lowest limit and maximal limit for sand ratio respectively.

d. The total consumption of gel material

\[ C_l \leq x_1 + x_4 \leq C_u. \]  

(17)

e. Additive consumption

\[ R_l \leq x_5 / (x_1 + x_4) \leq R_u \]  

(18)

where, \( x_5 / (x_1 + x_4) \) is the ratio of water reducing agent blend dosage and gel material dosage, and \( R_l, R_u \) are its lowest limit and maximal limit respectively.

f. The restraint of material volume

Assumed that the volume of concrete equals the sum of the absolute volume of each component and the volume of air contained in concrete, the amount of raw materials in every cube concrete requires to satisfy the following equation [10]

\[ \sum_{i=1}^{6} x_i / \rho_i + 10\alpha - 1000 = 0 \]  

(19)

where, \( \rho_i \) is the density of \( i \)th component; \( \alpha \) is the percentage of air in concrete and it equals 1 when air entraining agent not be used.

g. Cube compression strength

Admixture activity index was introduced to the calculation of mix proportions. Experimental investigation and the results of theoretical analysis show that the relationship of water gel ratio and concrete formulation strength presented by reference [11] possesses definite physical concept and better applicability, and can be applied to this optimum design. The relationship of water gel ratio and concrete formulation strength is as follows:

\[ A \cdot \left(-0.4952 + \frac{5.514 \times x_6}{x_1 + x_4}\right) \cdot \alpha_a \cdot f_{cc} \cdot \left(\frac{x_1 + x_4}{x_6} - \alpha_b\right) - f_{cu,k} + 1.645\sigma \geq 0 \]  

(20)

where \( A \) is mineral admixture activity index; \( f_{cu,k} \) is concrete cube compression strength; \( f_{cc} \) is cement strength; \( \sigma \) is standard deviation of concrete strength; \( \alpha_a, \alpha_b \) are regression coefficients.

### 3.3 Objective functions

The cooperating working ability of concrete and steel shape and the economic cost of concrete are chosen as the target of mix proportion optimization.

\( \tau_u \), which is ultimate bond strength on the interface of steel shape and concrete, characterizes the cooperating working ability of concrete and steel shape. According to formula (3) the objective function of ultimate bond strength can be established as follows

\[ \tau_u = \sum_{i=1}^{6} F_i (x_i) x_i \]  

(21)

where, \( x_i \) is the material factor of cement, aggregates, mineral admixture, etc.; \( F_i (x_i) \) is effect of material factor on ultimate bond strength, and the function relationship can be defined by equations (12) and (20).

The unit price of each material component are expressed as \( y_i \), the objective function of unit cost of concrete is as follows

\[ c = \sum_{i=1}^{6} y_i x_i. \]  

(22)

### 3.4 Solving process

The mix proportion optimization problem of ISHPC applied to SRC structures can be solved by using the method of sequential quadratic programming. It is the best means to solve the optimization problem with nonlinear objective function and nonlinear constrain function. Under the support of computer calculation speed, the means constructs quadratic programming sub-problem at every iteration point, and uses the solution of this sub-problem as iterative search direction to approach the solution of constrain optimization problem [12].

Typical inequality constrained optimization question is shown as

\[ \min c_0(x) \]  

s.t. \( c_i(x) \leq 0, \ i = 1, 2, \ldots, m \)  

(23)

where \( c_0(x) \) is objective function; \( x \) is design variables; \( c_i(x) \) is constraint function of state variables.
The following notation and definition are introduced for the current iterative point \( x^{(k)} \in \mathbb{R}^n \):

\[
I_k^- = \{ i \in I | c_i (x^{(k)}) \leq 0 \} \\
I_k^+ = \{ i \in I | c_i (x^{(k)}) > 0 \}
\]

(24)

\[
\varphi_k = \varphi (x^{(k)}) = \max \left\{ 0, c_i (x^{(k)}) | i \in I \right\} \\
= \max \left\{ 0, c_i (x^{(k)}) | i \in I_k^+ \right\}
\]

(25)

\[
D_i (x^{(k)}) = \left\{ \begin{array}{ll}
c_i (x^{(k)})^2, & i \in I_k^- \\
(\varphi (x^{(k)}) - c_i (x^{(k))))^2, & i \in I_k^+
\end{array} \right.
\]

(26)

\[
I (x^{(k)}) = \{ i \in I | D_i (x^{(k)}) = 0 \}.
\]

(27)

Accordingly the inequality constrained optimization can be translated into quadratic programming question:

\[
\min d^T \nabla c_0 (x^{(k)}) + \frac{1}{2} d^T B_k d
\]

s.t. \( c_i (x^{(k)}) + d^T \nabla c_i (x^{(k)}) \leq 0, \quad i \in I_k^- \)

(28)

\[
c_i (x^{(k)}) + d^T \nabla c_i (x^{(k)}) \leq \varphi_k, \quad i \in I_k^+
\]

where matrix \( B_k \) produces convergent descending direction, and \( d \) is principal searching direction, then the solution to question (28) will approach the solution to question (23).

Quadratic programming question has its feasible solution \( d = 0 \), assuming \( \left( d_0^{(k)} , \lambda^{(k+1)} \right) \) is its optimal solution.

Giving the following definition

\[
A_k = A (x^{(k)}) = (\nabla c_i (x^{(k)})), \quad i \in I
\]

\[
D_k = D (x^{(k)}) = (\text{diag} (D_i (x^{(k)})), \quad i \in I)
\]

(29)

\[
Q_k = Q (x^{(k)}) = \left( A_k^T A_k + D_k \right)^{-1} A_k^T
\]

\[
P_k = P (x^{(k)}) = E - A_k Q_k
\]

(30)

where \( E \) is identity matrix.

Constraint index set of quadratic programming question is \( L(k) \), and

\[
L_k = L (x^{(k)}) = L_k^+ \cup L_k^-
\]

(31)

\[
L_k^- = L^- (x^{(k)})
\]

\[
= \left\{ i \in I_k^- | c_i (x^{(k)}) + [\nabla c_i (x^{(k)})]^T d_0^{(k)} \right\}
\]

(32)

\[
L_k^+ = L^+ (x^{(k)})
\]

\[
= \left\{ i \in I_k^+ | c_i (x^{(k)}) + [\nabla c_i (x^{(k)})]^T d_0^{(k)} \right\}
\]

(33)

Quadratic programming question (28) can be generally solved in proper neighborhood of solution of inequality constrained optimization question (23), but the latter does not always have finite solution because \( B_k \) is not limited positive definite form, it is necessary to introduce a convergent subsidiary searching direction \( q^{(k)} \) [13–15].

\[
q^{(k)} = \rho^2 \left[ -P_k \nabla c_0 (x^{(k)}) + Q_k v^{(k)} \right], \quad \theta > 0
\]

(34)

\[
\rho_k = \rho (x^{(k)}) = \frac{\|P_k \nabla c_0 (x^{(k)})\|^2 + \omega_k + \varphi (x^{(k)})}{1 + \epsilon^T \pi^{(k)}}
\]

(35)

\[
\pi^{(k)} = \left\{ \pi^{(k)}_j, j \in I \right\} = -Q_k \nabla c_0 (x^{(k)})
\]

\[
\omega_k = \omega (x^{(k)}) = \sum_{j \in I} \max \left\{ -\pi^{(k)}_j, \pi^{(k)}_j D_j (x^{(k)}) \right\}
\]

(36)

Because matrix \( A_k^T A_k + D_k \) is positive definite form, the mentioned definition is of significance. If and only \( \rho_k = 0 \), \( x^{(k)} \) is solution to optimization question (23). If \( \left( d_0^{(k)} , \varphi_k \right) = (0, 0) \), then \( x^{(k)} \) is solution to quadratic programming question (28).

4 Example of mix proportion design

Design HSHPC applied to the SRC concrete structures. The strength grade range is from C60 to C100. Preferred cement is Chinese Qingling brand P.O 52.5R normal silicate cement. Preferred coarse aggregate is from Jingyang city, Shaanxi province, China; the particle size is 5 ~ 20 mm, and the density is 3.2 kg/m^3. Fine aggregate is coarse river sand from Ba river in Shaanxi province of China; the modulus of fineness is 2.8, and the density is 2.61 g/m^3. Admixture chooses polycarboxylate high efficiency water reducing agent, and the test of cement paste fluidity shows that it is better consistency with Qingling brand cement. Choosing fly ash and silicon ash as mineral additive, which are superfine particle and own high activity. Mixing water is drinking water.

In addition, Pull-out test is made to determine/compare ultimate bond strength of optimized
HSHPC and/or normal HSHPC (with steel shape). I-shaped cross-section steel in the specimens is composed of two number 10 channel and 6 mm thick steel plate. The thickness of covering layer is 60 mm, the anchorage length is 740 mm.

The optimum problem is solved by convergent sequence quadratic programming and the design program is compiled based on Matlab language [16, 17].

The comparison analysis of concrete unit cost and ultimate bond strength of optimized HSHPC with normal HSHPC is shown in Figures 3 and 4, and the test results of ultimate bond strength are given in Figure 4. It can be seen that the bond strength of optimized HSHPC and steel shape increases about 12%, and the results of theoretical analysis with experiment is approximately consistent; cost per cubic meter can be saved about 10% through optimization.

5 Conclusions

HSHPC mix proportion design discussed in this paper is a kind of method of multi-objective and nonlinear optimization. Using convergent sequence quadratic programming and based on Matlab language, the optimum problem can be solved satisfactorily. Optimized HSHPC can meet all performance requirements of SRC structures and have an obvious economic benefit.

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