Incorporating industrial constraints for multiobjective optimization of composite laminates using a GA

D.H. Bassir[1,3,a], F.X. Irisarri[2], J.F. Maire[2] and N. Carrere[2]

[1] Institut FEMTO-ST, UMR 6174, 24 rue de l'épitaphe, 25000 Besançon, France
[2] ONERA, DMSC 29 avenue de la division Leclerc, 92322 Chatillon Cedex, France
[3] Currently visiting Aerospace Structures, Faculty of Aerospace Engineering, TU Delft, The Netherlands

Abstract – In this article, complexities related to the multicriteria (multiobjective) optimization of laminated composite structures subjected to technological constraints will be presented. So, various technological constraints will be presented and a strategy of handling each constraint (in order to use the multi-objective optimization tools based on genetic algorithms) will be also introduced.

1 Introduction

An advantage of using fibre-reinforced composites over conventional materials is that they can easily reduce the mass of structures by adapting the stiffness to the requirements of many practical applications especially for aeronautic and aerospace structures. Berthelot, and Tsai and Hahn [1,2] reported on composites behaviour and the advantage of using such kind of materials. Also, many handbooks such as the Military Handbook [3] and the Aircraft Crash Survival Design Guide [4] are summerazing the theoretical and practical benchmark examples that help the designer to design basic structures with composite material. In these industrial fields the main goal is how to obtain the highest structural performances with substantial savings in terms of weight and stiffness, since composite materials possess high values of strength to weight and elasticity to weight ratios with respect to conventional materials as steel or aluminium alloys.

To improve the mechanical performances without increasing the mass of these structures, it is essential to apply an optimization process to define the optimal conception values. In general, design optimization of laminated composite structures involves not only one objective function but several objective functions [5, 6, 20] that are in permanent conflict. This kind of problems can be handled by a multicriteria approach, leading to designs which are balanced from an overall viewpoint. There exist two approaches: we can use either a direct method [7–10] or a posteriori method based on evolutionary strategies [11, 20]. Multicriteria optimization of composite structures has been discussed widely in the literature by various researchers, Adali et al. [6] presented multicriteria optimisation of laminated plates for maximum pre-buckling, buckling and post-buckling strength, Kere et al. [9] reported on multicriteria optimization

[a] Corresponding author: `hbassir@univ-fcomte.fr`

for strength design of composite laminates, while Adali et al. [10] used the multicriteria approach to design laminated cylindrical shells for maximum pressure and buckling load.

When we use the approach based on genetic algorithm (GA) to solve multicriteria optimization, the constraints are often hard to handle. Some authors such as Deb [11] have studied these complexities and gave an efficient and quick method to handle them and to obtain well-spread solutions through the Pareto front for nonlinear benchmark functions. In the next sections, we will focus on the complexities related to the optimization of laminated composite structures subjected to technological constraints that arise in the aeronautical fields and the strategy for handling each constraint in the NSGA-II program [11].

2 Multicriteria optimization

Multicriteria optimization problem is stated as follows: find the vector of design variables $\mathbf{x} = [x_1, x_2, x_3, ., x_n]^T$ which minimizes the vector of objective functions $F(x)$

$$\text{Min } \mathbf{F}(\mathbf{x}) = \text{Min}[f_1(x), f_2(x). ....f_k(x)]. \tag{1}$$

Subject to linear or nonlinear constraints:

$$\boldsymbol{g_j(x)} \geqslant 0 \quad j = 1, 2, \ldots., m.$$

The feasible domain (Fig. 1) defined by the constraints will be denoted by Ω. $f_i : \Omega \to R$, $i = 1, 2, ..., k$ are called criteria or objective functions and they represent the design objectives by which the performance of the laminate is measured in the case of composite material. A vector $\mathbf{x}^* \in \Omega$ is called Pareto-optimal solution if there is no vector $\mathbf{x} \in \Omega$ which would decrease some criteria without
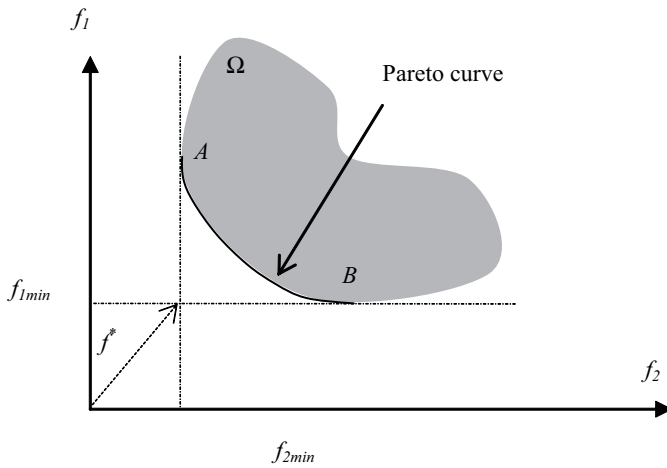
Fig. 1. Pareto curve (Min (f_1 and f_2)).

Fig. 2. Direct (a) and posteriori (b) methods for multicriteria optimisation.

weighting method which reduces a vector-optimization problem to a scalar optimization problem where, for instance, the scalar objective function $f(x)$ can be defined as the weighted sum of the individual objective functions. The Pareto-optimal solutions obtained by this method depends on the choice of the weighting factors α_i that will generate a unique Pareto optimal solution of the original problem. This approach needs to define appropriate weighting factors that will guide the convergence of the optimal design process [7]. However, this definition depends on the decision-maker and the shape of the Pareto domain (continuity, convexity and the number of limitations to handle). If the weighting factors are not well chosen, this approach can converge in local zone of the Pareto domain especially when the optimal solution is very sensitive to the weighting factors. This conclusion was described and demonstrated for some academic examples by different authors such as Das [12]. Other researchers have defined good strategies to calculate these weighting factors using the sensitivities evaluation of the objective functions [13] which turns out to be problematic in the case of the non differential functions. Moreover, when the final solution is reached, there is no guaranty that this solution is the best one for the decision-maker as he needs to run the process many times before making his decision.

The posteriori method consists in starting with a group of initial solutions that we spray uniformly to have a global idea about the Pareto front and to make the final decision easier to take for the decision-maker. In the case of composite materials, GAs are considered among the most efficient algorithms to solve the optimization problem. The theoretical foundations of GAs were first introduced by Holland and then extended by Goldberg [14,15]. This method uses the evolutionary survival-of-the fittest optimization mechanism. The principle of GAs is to simulate the evolution of one population of individuals to which we apply different production operators (*selection, crossover and mutation*). Many researchers have been inspired by the principles of the GA to use them with some modifications to solve their optimization problems either with one or more objective functions with some basic constraints [16,17].

There are two main issues in using GAs. Firstly, the comparison between two solutions can not be achieved easily in the selection process. Secondly, the constraints handling is difficult to represent as an inequality or equality equation. To overcome these difficulties, many researchers like Deb for instance, proposed approaches based on their previous investigations on GAs [18,19]. Deb has introduced in his current algorithm NSGA-II the idea of non-dominated sorting to speed up the convergence and increase the performance of his previous algorithm NSGA. To choose between two solutions i and j to continue the iterative process of the GA a non-dominate sorting approach in the selection process is used as follows:

- between two admissible solutions i and j, choose the one with the best objective function;
- if solution i is admissible and j is not, choose solution i;

causing a simultaneous increase in at least one criteria function. Usually several Pareto-optimal solutions exist.

The optimal solutions from the Pareto domain can be reached by two main strategies: either by a direct approach or by a posteriori approach (Fig. 2).

The direct method is based on the transformation of the initial problem into a single optimization problem. This transformation can be done by using for example the
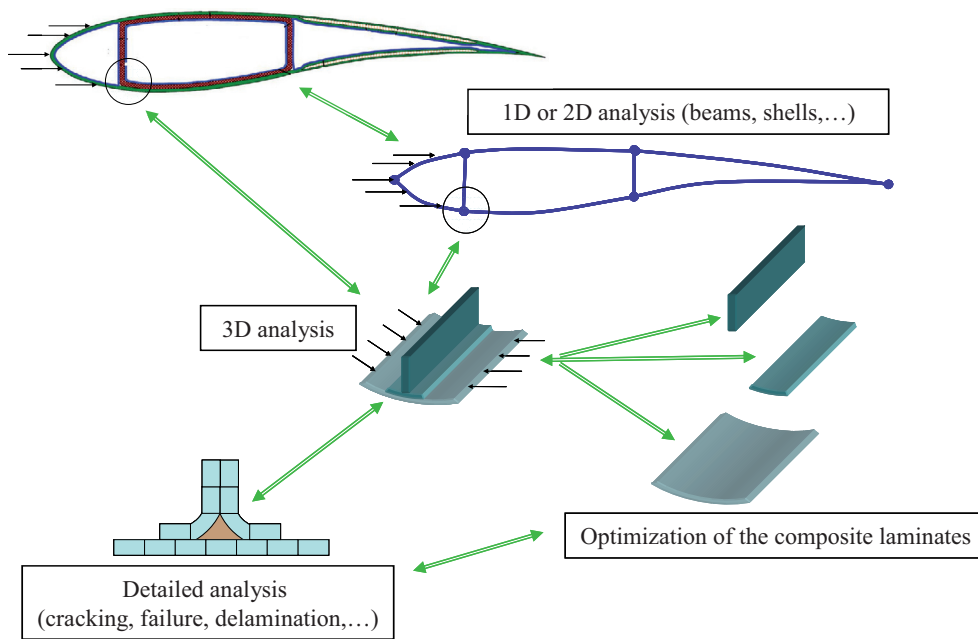
Fig. 3. Multilevel strategy for structural optimisation in the aeronautical industry.

- if the solutions i and j are not admissible, choose the one with the minimum violation of the constraints.

Next sections will be devoted to the particularities and the origin of the constraints to better understand them and find the right way to handle them as constraints in NSGA-II. Procedure as a pseudo code of these constraints will also be presented.

To handle the constraints in the GA program we define the following composite fitness function for any solution x such as: $F(x) = f(x)$ if x is feasible, otherwise $F(x) = f_{max} + CV(x)$. f_{max} is the objective function value of the worst feasible solution in the population and $CV(x)$ is the overall normalized constraint violation of the solution x. Thus, there is no need to have any penalty parameter for handling the constraints as usually used in the common approaches. Constraints are normalized to avoid scaling problems and are equal to one in the case of feasible solution.

3 Constraints handling for laminated composite structures

In the aeronautical industry, structural optimization process is often based on the multilevel strategy that can be built as a pyramid (Fig. 3).

The global optimization process starts by the optimization of the basic components. Once the validation of each component is established, the optimization process is applied to the upper level until the final design of the whole structure is reached. Nevertheless an iterative scheme is often required because of load redistribution problems. In the following, we will focus on the industrial constraints for the detailed design of the laminated components of the structure.

In the design of laminated composite structures, the major objective is to find a laminate lay-up configuration that satisfies several requirements or conception rules. In practice, the design starts by the choice of the number of plies. The stacking sequence design is considered in a second time. From experience of manufacturing composite laminates, some rules appeared for choosing the stacking sequences. Nowadays, these rules constitute a group of constraints that increase the difficulty for laminated composite optimization. Among these rules:

1. laminate must be symmetric with respect to its mid-plane;
2. laminate must be balanced regarding to main direction of the loads;
3. maximum number of plies in one group;
4. maximal disorientation between two consecutive layers;
5. the stacking sequence must be homogeneous;
6. minimum amount of plies in each direction;
7. use of integer orientation.

3.1 Symmetric stacking

The symmetry about the mid-plane eliminates the membrane/plate couplings. In terms of manufacturing, this rule of conception cancels the twisting of the panels during their elaboration. The modelling of this rule consist in taking in the account for a vector x $(\theta_1, \theta_2, \ldots \theta_N)$ only the upper or lower part. It means that we keep only $N/2$ variable from the vector x. The other part is calculated by symmetry.

3.2 Balanced stacking

This rule requires to have the same number of the plies $+\theta$ or $-\theta$, where $\theta \in [0, 90]$. It eliminates the membrane coupling between shear and traction. This rule can be integrated by simply counting the number of the $+\theta$ and the $-\theta$ than we create one limitation associated for each angle such as

$$\begin{aligned} \boldsymbol{Begin} \quad & \{\theta = \theta_j \in [0, 90]; \\ & \boldsymbol{Do} \ (i = 1 \to i = NP) \\ & \{if \ \theta_i = +\theta_j \ than \ N_{+\theta_j} + + \ ; \\ & \quad if \ \theta_i = -\theta_j \ than \ N_{-\theta_j} + + \\ & ;\} \\ & T_j = ||N_{+\theta_j} - N_{-\theta_j}||; \} \\ & \boldsymbol{If} \ (count \geqslant 1) \ g_{\theta_i}(x) = -\sum T_j; \\ & \boldsymbol{Else} \ g_{\theta_i}(x) = 1.0; \end{aligned}$$

NP is the number of plies or layers used in the laminated composite structure.

3.3 Maximum number of plies in one group

This rule consists in minimizing the number consecutive plies in a group of the same orientation. Depending on the ply thickness, the usual limit is fixed to 3 or 4 plies with the same orientation together. The Military Handbook recommends two different limits in terms of the total thickness depending on the orientations to the edge of the panels. The first limit is generally fixed to 0.8 mm when the orientation is parallel to the free edges. The second limit is fixed to 0.38 mm when the orientation is perpendicularly to the free edges. This rule aims at minimizing problems related to matrix cracking, such as the shear-out failure mode in bolted joints. This rule improves also the composite behaviour against impacts. It can be integrated as described in the following pseudo code.

$$\begin{aligned} \boldsymbol{Begin} \quad & \{test \ (NP \geqslant NPG); \\ & \boldsymbol{Do} \ (j = 1 \to j = NP) \\ & \quad \{\boldsymbol{Do} \ (i = j \to i = NPG) \\ & \quad \boldsymbol{If} \ (\sum ||(\theta_i - \theta_{i-1})|| \approx 0) \\ & \quad than \ count \ + +; \} \\ & \} \ \boldsymbol{If} \ (count \ \geqslant 1) \ than \ g_{\theta i}(x) = -count; \\ & \boldsymbol{Else} \ g_{\theta i}(x) = 1.0; \\ & \boldsymbol{End} \end{aligned}$$

NPG is the maximum number of plies in a group.

3.4 Maximum disorientation

The rule of disorientation imposes a maximal disorientation of 45 degrees between two successive plies. The

aim of this conception rule is to minimize interlaminar shear effects and reduce delamination problems, especially around holes and free edges. In this perspective, the pertinence of this rule can be evaluated with the help of appropriated numerical tools to get the stress values around holes and the free edges. The pseudo code to describe this rule can be as follows:

$$\begin{aligned} \boldsymbol{Begin} \quad & \{\boldsymbol{Do}(i = 1 \to i = NP) \\ & \{\boldsymbol{If} \ (||(\theta_i - \theta_{i-1})|| \geqslant 45) \ \boldsymbol{than} \ (count + +; \\ & T_i = ||(\theta_i - \theta_{i-1})||/45;)\} \\ & \}\boldsymbol{If} \ (count \neq 0) \ \boldsymbol{than} \ g_\theta(x) = \\ & \quad\quad -\sum T_i/NP; \\ & \boldsymbol{Else} \ g_\theta(x) = 1.0; \\ & \boldsymbol{End} \end{aligned}$$

3.5 Homogeneous stacking sequences

This rule consists in distributing the different orientations along the thicknesses with respect to the other rules. This constraint is particularly hard to handle in the case of panel optimization for buckling. The principal objective of this rule is to reduce the twisting/bending coupling effects, particularly influents in the buckling of panels under shearing loads. The problem appears because engineers still do not master the conception in term of shearing and ignore most often the sign of this shearing.

3.6 Minimum percentage for each angle used in the stacking sequence

This rule imposes for instance a minimum of 10% of orientation for each $0°$, $\pm45°$ and $90°$ orientations. The aim of this rule is to avoid obtaining a composite laminate whose behaviour is governed by the matrix in some directions (nonlinear case and creeps). In reality, it is mainly justified by the ignorance of the loadings and the necessity to insure a minimal stiffness and strength in all the directions. Therfore this rule can be reformulated in terms of minimum stiffness in each direction. Actually, the value of 8% is already employed for some aeronautic applications. The pseudo code to describe this rule can be as follows:

$$\begin{aligned} \boldsymbol{Begin} \quad & \{\boldsymbol{Do}(i = 1 \to i = NP) \\ & g_{\theta i}(x) = (\theta_i \cdot 100/ \ (Number \ of \ \theta_j)) \\ & \boldsymbol{Than} \ (count + +; T_i = ||(\theta_i - \theta_{i-1})||/45;)\} \\ & g_{\theta i}(x) = (g_{\theta i}(x) - PAng)/PAng; \\ & \boldsymbol{End} \end{aligned}$$

PAng is the percent imposes for angles.

3.7 Use of integer orientation

The use of only specific orientations such as 0, ±45 and 90 seems to be no longer justified, as we are actually capable of manufacturing all kind of orientations. This rule
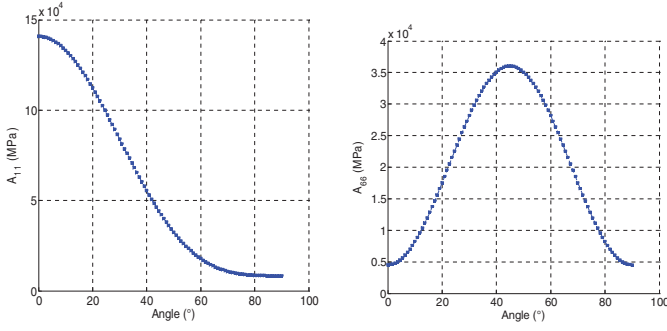
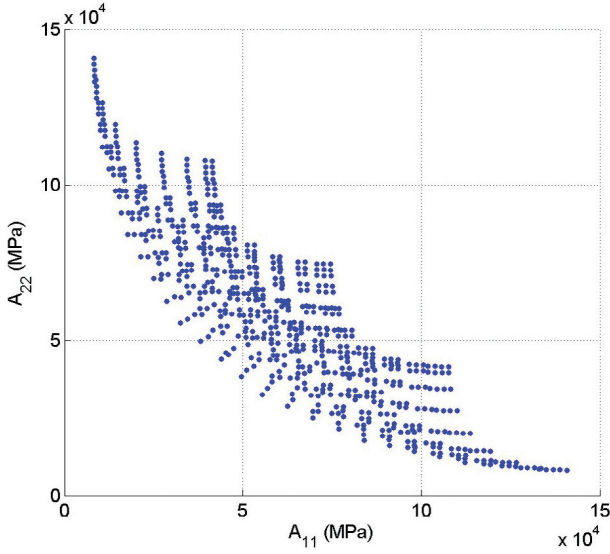**Fig. 4.** A11 and A66 in function of the orientation angle.

Fig. 5. Feasible domain in the case of a maximization of A11and A22 for 4 staking of 1 mm.

comes probably from the decomposition in elementary efforts of longitudinal and transverse tension/compression and shear that correspond to principal axes of the solicitations. In the case of a mixed variable the NSGA-II allows such combination (integer, real or mixed variables) [11,18,19].

4 Numerical application to composite beam

To underline the complexity related to the multi-objective optimization in laminated composite structures we consider a basic composite beam with the classical laminate theory with constant thickness. In this case, we have the following $[ABD]$ matrix such as:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ \rho \end{bmatrix} \qquad (2)$$

where: $[A]$, $[D]$ and $[B]$ are respectively the membrane, bending and coupling terms of the stiffness matrix; (N, M) are the in-plane and out-plane internal forces; (ε, ρ) are respectively the in-plane and out-plane strains.

The objective functions that can be considered in the optimization process are the stiffness components A_{ij}, B_{ij}
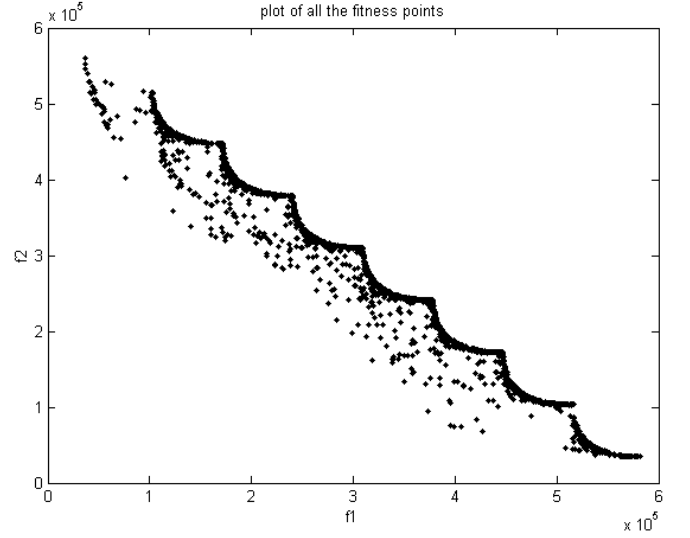

Fig. 6. Feasible domain for maximization problem of A11 (F_1) and A22 (F_2) for 8 staking.
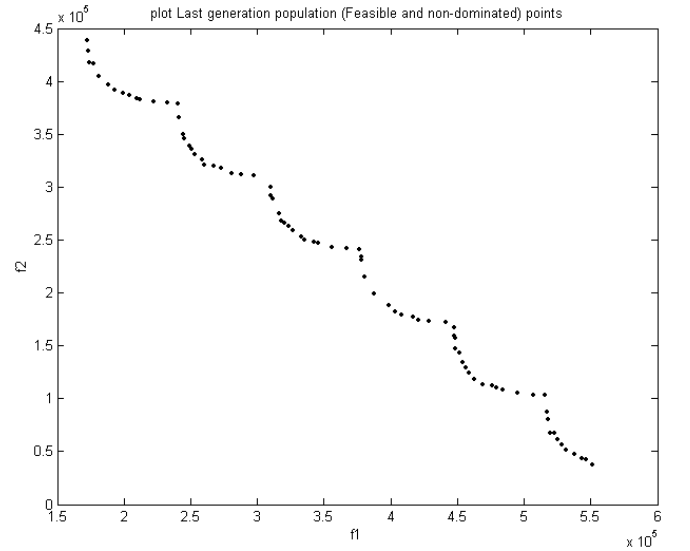

Fig. 7. Pareto Front at the convergence for maximization problem of A11 (F_1) and A22 (F_2) for 8 staking.

and D_{ij}. These functions are in general non-linear functions of the orientation angle (Fig. 4). In the following we consider a composed beam with the following material T700/M21 (E11 = 140GP, E22 = 8.2GP, υ12 = 0.3, G12 = 4.5GP).

If we consider for example the maximisation of (A11, A22) with $\theta_{(i=1-4)} \in [-\pi/2, \pi/2]$ the feasible domain is described in (Fig. 5).

In the case of the 8 staking (Fig. 6) we note clearly the increase of the non linearity, the discontinuity and the non convexity. At the convergence, the Pareto front is described in Figure 7. We can note that the final solutions are uniformly distributed along the Pareto front. This distribution is generally hard to obtain with common evolutionary strategies. If we integrate for example the following constraints (A11 must be greater than 350 GPa.mm
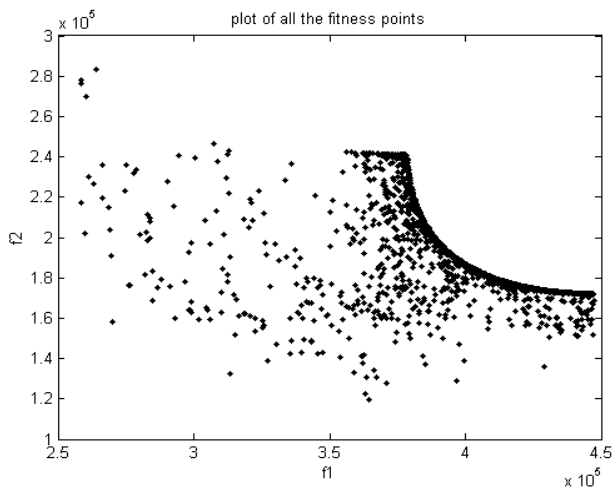
Fig. 8. Feasible domain with constraints on A11 (F_1) and A22 (F_2).
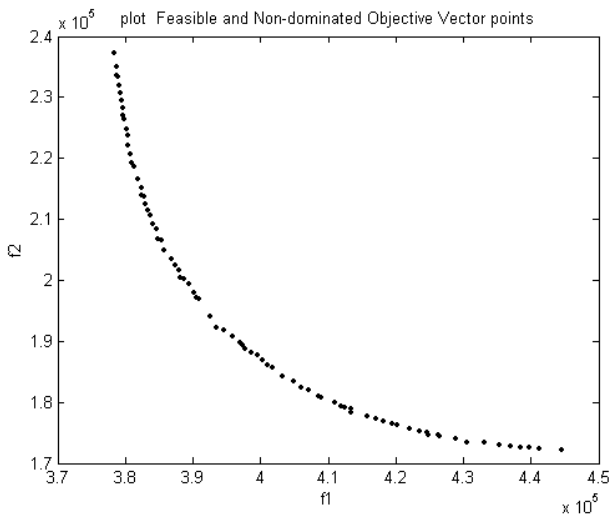

Fig. 9. Feasible domain with constraints on A11 and A22 with symetry stacking.

and A22 must be greater than 150 GPa.mm) we obtain the feasible domain in Figure 8. This domain is reduced in the case of symmetrical orientation Figure 9.

5 Conclusions

In this article, we have underlined the difficulties related to the multi-objective optimization of laminated composite under technological constraints. The handling method used in *NSGA-II* program has demonstrated its efficiency for a beam example to obtain a uniform spray of the Pareto solutions. However, further research is to be done concerning the integration of new technological constraints for composite materials.

References

1. J.M. Berthelot, *Matériaux composites: comportement mécanique et analyse des structures* (Masson, Paris, 1992)
2. S.W. Tsai, T. Hahn, *Introduction of composite materials* (Technomic Publ., Lancaster, 1980)
3. *Military Hand Book "MIL-HDBK-17-3E", Working Draft*, Design and Analysis, 23 Jan. 1997, Chap. 4
4. *Aircraft Crash Survival Design Guide*, USAAVSCOM TR 89-D-22A-E, Vol. I-IV, 1989
5. *Multicriteria Design Optimization*, edited by H. Eschenauer, J. Koski, A. Osyczka (Springer-Verlag, 1990)
6. S. Adali, M. Walker, V.E. Verijenko. Compos. Struct. **35**, 117 (1996)
7. M. Domaszewski, D.H. Bassir, W.H. Zhang, *Stress displacement and weight minimization by multicriteria optimization and game theory approach*, Computer Aided Optimum Design of Structures OPTI99 (Orlando, Florida, USA, WIT Press, 1999), pp. 171–181
8. R. Spallino, S. Rizzo. Mechanics Research Communications **29**, (2002)
9. P. Kere, M. Lyly, J. Koski. Compos. Struct. **62**, 329 (2003)
10. S. Adali, A. Richter, V.E. Verijenko. Microcomput. Civil Eng. **10**, 269 (1995)
11. K. Deb, A. Patrap, S. Agarwal, T. Meyarivan, *A fast and Elitist multi-objective Genetic algorithm: NSGA-II*, Technical Report No. 200001, 2005, Kanpur Genetic Algorithms Laboratory, India Institute of Technology
12. I. Das, Non linear multicriteria optimization and robust optimality, Ph.D. Thesis (Rice University, Houston, USA, 1997)
13. W.H. Zhang, M. Domaszwski, C. Fleury. Int. J. Numer. Meth. Eng. **52**, 889
14. J.H. Holland, *Adaptation in natural and artificial systems* (University of Michigan Press, Ann Arbor, 1975)
15. D.E. Goldberg, *Genetic algorithms in search, optimisation, and machine learning* (New York, Addison-Wesley, 1989)
16. Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolution* (Springer-Verlag, Heidelberg, 1994)
17. C.M. Fonceca, P.J. Fleming, *Multiobjective optimization and multiple constraint handling with evolutionary algorithms*, part II: Application example, IEEE transactions on systems, man and Cybernetics: Part A: systems and humans, 1998, pp. 38–47
18. K. Deb, D.E. Goldberg, *An investigation of niche and species formation in genetic function optimization*, in *Proceedings of the Third International Conference on Genetic Algorithms*, edited by J.D. Schaffer (San Mateo, 1989), pp. 42–50
19. K. Deb, R.B. Agrawal, *Simulated binary crossover for continuous search space* (Complex Systems, 1995), pp. 115–148
20. F.X. Irisarri, D.H. Bassir, J.F. Maire, N. Carrere, Multiobjective stacking sequence optimisation strategy for laminated composite structures, *First International conference on multidisciplinary optimization and Applications*, April 17–20, 2007 (EDP Sciences), ISBN 978-2-7598-0023-0
21. K. Deb. International Journal of Simulation and Multidisciplinary Design Optimization **1**, 1 (2007)