

Current trends in evolutionary multi-objective optimization

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Abstract – In a short span of about 14 years, evolutionary multi-objective optimization (EMO) has established itself as a mature field of research and application with an extensive literature, many commercial softwares, numerous freely downloadable codes, a dedicated biannual conference running successfully four times so far since 2001, special sessions and workshops held at all major evolutionary computing conferences, and full-time researchers from universities and industries from all around the globe. In this paper, we make a brief outline of EMO principles, some EMO algorithms, and focus on current research and application potential of EMO. Besides, simply finding a set of Pareto-optimal solutions, EMO research has now diversified in hybridizing its search with multi-criterion decision-making tools to arrive at a single preferred solution, in utilizing EMO principle in solving different kinds of single-objective optimization problems efficiently, and in various interesting application domains which were not possible to be solved adequately due to the lack of a suitable solution technique.

1 Introduction

Evolutionary optimization (EO) methodologies are now popularly used in various problem solving tasks involving nonlinearities, large dimensionality, non-differentiable functions, non-convexity, multiple optima, multiple objectives, uncertainties in decision and problem parameters, large computational overheads, and various other complexities for which the classical optimization methodologies are known to be vulnerable. In a recent survey conducted before the World Congress on Computational Intelligence (WCCI) in Vancouver 2006, the evolutionary multi-objective optimization (EMO) field was judged as one of the three fastest growing field of research and application among all computational intelligence topics. For solving single-objective optimization problems or in other tasks focusing to find a single optimal solution, the use of a population of solutions in each iteration may at first seem like an overkill; in solving multi-objective optimization problems an EO procedure is a perfect fit [13]. The multi-objective optimization problems, by theory, give rise to a set of Pareto-optimal solutions which need a further processing to arrive at a single preferred solution. To achieve the first task, it becomes quite a natural proposition to use a modified EO, because the use of population in an iteration helps an EO to simultaneously find multiple Pareto-optimal solutions in a single simulation run.

In this paper, we briefly describe the principles of EMO and then discuss a few well-known computational procedures. Thereafter, we highlight the current interest

of research and application of EMO. It is clear from the discussions that EMO is not only being found to be useful in solving multi-objective optimization problems, it is also helping to solve other kinds of optimization problems in a manner better than they are traditionally solved. As a by-product, EMO-based solutions are helping to reveal important hidden knowledge about a problem – a matter which is difficult to achieve otherwise. This paper should motivate interested readers to look into the extensive EMO literature indicated in the reference list and in appendix for more details.

2 Evolutionary Multi-objective Optimization (EMO)

As in the single-objective optimization problem, the multi-objective optimization problem usually has a number of constraints which any feasible solution (including the optimal solution) must satisfy:

$$\left. \begin{array}{l} \text{Minimize/Maximize } f_m(\mathbf{x}), \quad m = 1, 2, \dots, M; \\ \text{subject to } g_j(\mathbf{x}) \geq 0, \quad j = 1, 2, \dots, J; \\ h_k(\mathbf{x}) = 0, \quad k = 1, 2, \dots, K; \\ x_i^{(L)} \leq x_i \leq x_i^{(U)}, \quad i = 1, 2, \dots, n. \end{array} \right\} \quad (1)$$

A solution \mathbf{x} is a vector of n decision variables: $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$. The last set of constraints are called variable bounds, restricting each decision variable x_i to take a value within a lower $x_i^{(L)}$ and an upper $x_i^{(U)}$ bound.

Different solutions may produce trade-offs (conflicting scenarios) among different objectives. A solution that is extreme (in a better sense) with respect to one objective requires a compromise in other objectives. This prohibits

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one to choose a solution which is optimal with respect to only one objective. This clearly introduces two main goals of multi-objective optimization:

1. Find a set of solutions close to the optimal solutions, and
2. Find a set of solutions which are diverse enough to represent the true spread of optimal solutions.

Evolutionary multi-objective optimization (EMO) algorithms follow both the above principles more directly than their existing counterparts.

From a practical standpoint, a user needs only one solution, no matter whether the associated optimization problem is single-objective or multi-objective. In the case of multi-objective optimization, the user is now in a dilemma. Since a number of solutions are optimal, the obvious question arises: Which of these optimal solutions must one choose? This is not an easy question to answer. It involves many higher-level information which are often non-technical, qualitative and experience-driven. However, if a set of many trade-off solutions are already worked out or available, one can evaluate the pros and cons of each of these solutions based on all such non-technical and qualitative, yet still important, considerations and compare them to make a choice. Thus, in a multi-objective optimization, ideally the effort must be made in finding the set of trade-off optimal solutions by considering all objectives to be important. After a set of such trade-off solutions are found, a user can then use higher-level qualitative considerations to make a choice. In view of these discussions, the following principles are used in evolutionary multi-objective optimization (EMO) procedures:

- Step 1 Find multiple trade-off optimal solutions with a wide range of values for objectives.
- Step 2 Choose one of the obtained solutions using higher-level information.

3 State-of-the-art EMO methodologies

A number of non-elitist EMO methodologies [23, 26, 39] gave a good head-start to the research and application of EMO, but suffered from the fact that they did not use an important operator – an elite-preservation mechanism – in their procedures. An addition of elitism in an EO provides a monotonically non-degrading performance. The next-level EMO algorithms implemented an elite-preserving operator in different ways and gave birth to elitist EMO procedures, some of which we describe in the following subsections.

3.1 Elitist non-dominated sorting GA or NSGA-II

The NSGA-II procedure [14] for finding multiple Pareto-optimal solutions in a multi-objective optimization problem has the following three features: (i) it uses an elitist principle, (ii) it uses an explicit diversity preserving mechanism, and (iii) it emphasizes non-dominated solutions. In

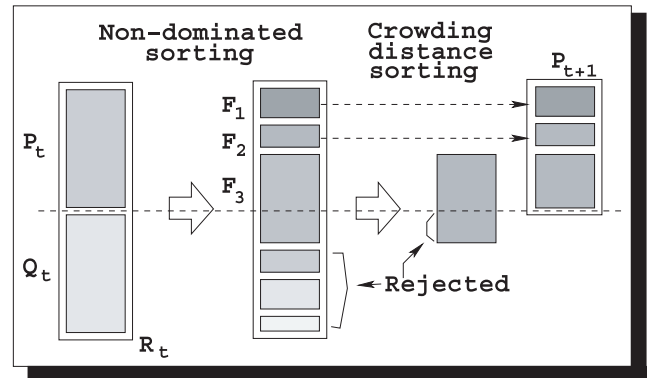


Fig. 1. Schematic of the NSGA-II procedure.

NSGA-II, the offspring population Q_t is first created by using the parent population P_t and the usual evolutionary operators. Thereafter, the two populations are combined together to form R_t of size $2N$. Then, a non-dominated sorting is used to classify the entire population R_t . Once the non-dominated sorting is over, the new population is filled by solutions of different non-dominated fronts, one at a time. The filling starts with the best non-dominated front and continues with solutions of the second non-dominated front, followed by the third non-dominated front, and so on. Since the overall population size of R_t is $2N$, not all fronts can be accommodated in N slots available in the new population. All fronts which could not be accommodated are simply deleted. When the last allowed front is being considered, there may exist more solutions in the last front than the remaining slots in the new population. This scenario is illustrated in Figure 1. Instead of arbitrarily discarding some members from the last front, the solutions which will make the diversity of the selected solutions the highest are chosen.

Due to the simplicity and efficient usage of its operators, NSGA-II procedure is probably the most popular EMO procedure today. A C code implementing the procedure is available from author's web site <http://www.iitk.ac.in/kangal/soft.htm>.

3.2 Strength Pareto EA (SPEA) and SPEA2

Zitzler and Thiele [44] suggested an elitist multi-criterion EA with the concept of non-domination in their strength Pareto EA (SPEA). They suggested maintaining an external population at every generation storing all non-dominated solutions discovered so far beginning from the initial population. This external population participates in evolutionary operations. At each generation, a combined population with the external and the current population is first constructed. All non-dominated solutions in the combined population are assigned a fitness based on the number of solutions they dominate. To maintain diversity and in the context of minimizing the fitness function, they assigned a higher fitness value to a non-dominated solution having more dominated solutions in the combined population. On the other hand, a higher

fitness is also assigned to solutions dominated by more solutions in the combined population. Care is taken to assign no non-dominated solution a fitness worse than that of the best dominated solution. This assignment of fitness makes sure that the search is directed towards the non-dominated solutions and simultaneously diversity among dominated and non-dominated solution are maintained. On knapsack problems, they have reported better results than other methods used in that study.

In their subsequent improved version (SPEA2) [43], three changes have been made. First, the archive size is always kept fixed (thus if there are fewer non-dominated solutions in the archive than the predefined archive size, dominated solutions from the EA population are copied to the archive). Second, a fine-grained fitness assignment strategy is used in which fitness assignment to the dominated solutions are slightly different and a density information is used to resolve the tie between solutions having identical fitness values. Third, a modified clustering algorithm is used with k -th nearest neighbor distance measure and special attention is made to preserve the boundary elements.

3.3 Pareto Archived ES (PAES) and Pareto Envelope based Selection Algorithms (PESA and PESA2)

Knowles and Corne [28] suggested a simple possible EMO using evolution strategy (ES). In their Pareto-archived ES (PAES) with one parent and one child, the child is compared with respect to the parent. If the child dominates the parent, the child is accepted as the next parent and the iteration continues. On the other hand, if the parent dominates the child, the child is discarded and a new mutated solution (a new child) is found. However, if the child and the parent do not dominate each other, the choice of child or a parent considers the second task of keeping diversity among obtained solutions using a crowding procedure. To maintain diversity, an archive of non-dominated solutions found so far is maintained. The child is compared with the archive to check if it dominates any member of the archive. If yes, the child is accepted as the new parent and the dominated solution is eliminated from the archive. If the child does not dominate any member of the archive, both parent and child are checked for their *nearness* with the solutions of the archive. If the child resides in a least crowded region in the parameter space among the members of the archive, it is accepted as a parent and a copy of added to the archive. It is interesting to note that both features of (i) emphasizing non-dominated solutions, and (ii) maintaining diversity among non-dominated solutions are present in this simple algorithm. Later, they suggested a multi-parent PAES with similar principles as above.

In their subsequent version, they called Pareto Envelope based Selection Algorithm (PESA) [9], they combined good aspects of SPEA and PAES. Like SPEA, PESA carries two populations (an EA population and a comparatively larger archive population). Non-

domination and the PAES's crowding concept is used to update the archive with the newly created child solutions.

In an extended version of PESA (or PESA2) [10], instead of applying the selection procedure on population members, hyperboxes in the objective space are selected based on the number of residing solutions in the hyperboxes. After hyperboxes are selected, a random solution from the chosen hyperboxes is kept. This region-based selection procedure has shown to perform better than individual-based selection procedure of PESA. In some sense, PESA2 selection scheme is similar in concept to the ϵ -dominance in which predefined ϵ values determine the hyperbox dimensions. Other ϵ -dominance based EMO procedures [18, 30] have shown to be computationally faster and to better distribute solutions than NSGA-II or SPEA2.

There also exist other competent EMOs, such as multi-objective messy GA (MOMGA) [41], multi-objective micro-GA [6], neighborhood constraint GA [31], ARMOGA [36], and others. Besides, there exists other EA based methodologies, such as particle swarm EMO [7, 35], ant-based EMO [24, 33], and differential evolution based EMO [1].

4 Constraint handling in EMO

The constraint handling method modifies the domination principle. In the presence of constraints, each solution can be either feasible or infeasible. Thus, there may be at most three situations: (i) both solutions are feasible, (ii) one is feasible and other is not, and (iii) both are infeasible. We consider each case by simply redefining the domination principle as follows. A solution $\mathbf{x}^{(i)}$ is said to 'constrain-dominate' a solution $\mathbf{x}^{(j)}$, if any of the following conditions are true: (i) solution $\mathbf{x}^{(i)}$ is feasible and solution $\mathbf{x}^{(j)}$ is not, or (ii) solutions $\mathbf{x}^{(i)}$ and $\mathbf{x}^{(j)}$ are both infeasible, but solution $\mathbf{x}^{(i)}$ has a smaller constraint violation, or (iii) solutions $\mathbf{x}^{(i)}$ and $\mathbf{x}^{(j)}$ are feasible and solution $\mathbf{x}^{(i)}$ dominates solution $\mathbf{x}^{(j)}$ in the usual sense. The above change in the definition requires a minimal change in the NSGA-II or other EMO procedures described earlier. Figure 2 shows the non-dominated fronts on a six-membered population due to the introduction of two constraints. In the absence of the constraints, the non-dominated fronts (shown by dashed lines) would have been $((1, 3, 5), (2, 6), (4))$, but in their presence, the new fronts are $((4, 5), (6), (2), (1), (3))$. The first non-dominated front is constituted with the best infeasible solutions in the population and any feasible solution lies on a better non-dominated front than an feasible solution. This simple modification in domination principle allows to form an appropriate hierarchy among solutions in the population for them to move towards the true constrained Pareto-optimal front.

5 Current EMO research and practices

With the fast development of efficient EMO procedures, availability of free and commercial softwares (some of

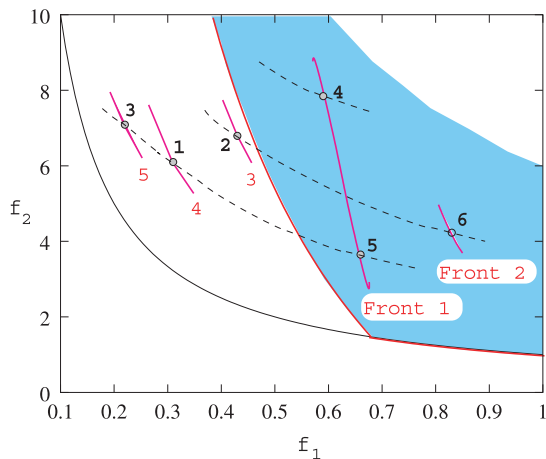


Fig. 2. Non-constrain-dominated fronts.

which are described in the Appendix), and applications to a wide variety of problems, EMO research and application is in its peak at the current time. It is difficult to cover every aspect of current research in a single paper. Here, we outline three main broad areas of research and application.

5.1 EMO and decision-making

It is important here to note that finding a set of Pareto-optimal solutions by using an EMO is only a part of multi-objective optimization, as choosing a particular solution for implementation is the remaining decision-making task which is also an equally important task. In the view of the author, the decision-making task can be considered from two main aspects:

1. **Generic consideration:** There are some aspects which most practical users would like to use in narrowing down their choice. For example, in the presence of uncertainties in decision variables and/or problem parameters, the users are usually interested in finding *robust* solutions which are relatively insensitive to the variation in decision variables or parameters. In the presence of such variations, no one is interested in Pareto-optimal but sensitive solutions. Practitioners do not hesitate sacrificing globally optimal solutions in order to achieve robust solutions which lie on relatively flat part of the objective function or away from the constraint boundaries. In such scenarios, instead of finding the globally Pareto-optimal front, the user may be interested in finding the robust frontier which may be partially or totally different from the true Pareto-optimal front. A robust EMO procedure [15] is shown to find only those Pareto-optimal solutions or solutions close to them which are robust and insensitive to parameter and variable perturbations. In addition, instead of finding the entire Pareto-optimal front, the users may be interested in finding some specific solutions on the Pareto-optimal front, such as *knee* points (requiring a large sacrifice in at least one objective to achieve a small gain in another),

points having correlated relationship between objectives to decision variables (identifying the portion of the Pareto-optimal front which possesses simpler relationship between objective values and decision variable values), points having multiplicity (finding Pareto-optimal solutions corresponding to multiple (say at least two or more) decision variable vectors but each having identical objective values), points for which decision variable values are well within their allowable bounds and not near their lower or upper boundaries, points having some theoretical aspects such as all Lagrange multipliers having more or less identical magnitude (condition for having equal importance to each constraint), and others. These considerations are motivated from the fundamental and practical aspects of optimization and may be applied to most multi-objective problem solving tasks, without any consent of a decision-maker.

2. **Subjective consideration:** In this category, any problem-specific information can be used to narrow down the choices and the process may even lead to a single preferred solution at the end. Most decision-making procedures use some preference information (utility functions, reference point approaches [42], reference direction approaches [29], marginal rate of return and a host of other considerations [34]) to select a subset of Pareto-optimal solutions. A recent book is dedicated to the discussions of many such multiple criteria decision making (MCDM) tools and collaborative suggestions of using EMO with such MCDM tools [4]. Some hybrid EMO and MCDM algorithms are also suggested in the recent past [16, 17, 22, 32, 40].

5.2 Multi-objectivization

Interestingly, the act of finding multiple Pareto-optimal solutions using an EMO procedure has found its application outside the realm of solving multi-objective optimization problems per se. The concept of finding optimal trade-off solutions are applied to solve other kinds of optimization problems as well. For example, the EMO concept is used to solve constrained single-objective optimization problems by converting the task as a two-objective optimization task of additionally minimizing an aggregate constraint violation [8]. This eliminated the need of having any penalty parameter. If viewed this way, the usual penalty function approach used in the classical optimization studies is a special weighted-sum approach to the bi-objective optimization problem of minimizing the objective function and minimizing the constraint violation, for which the weight vector is a function of penalty parameter. The reduction of a well-known difficulty in the genetic programming studies, called the ‘bloating’, by minimizing the size of programs as an additional objective helped find high-performing solutions with a smaller size of the code [2]. Minimizing the intra-cluster distance and maximizing inter-cluster distance simultaneously in a bi-objective formulation of a clustering problem is found to yield better solutions than the usual single-objective

minimization of the ratio of the intra-cluster distance to the inter-cluster distance [25]. A recent edited book [27] describes many such interesting applications in which EMO methodologies have help shown problems which are otherwise (or traditionally) not treated as multi-objective optimization problems.

5.3 Applications

EMO methodologies are being and must be applied to more interesting real-world problems to demonstrate the utility of finding multiple trade-off solutions. Although some recent studies are finding that EMO procedures are not computationally efficient to find multiple and widely distributed set of solutions on problems having a large number of objectives (more than five objectives or so) [11, 20], EMO procedures are still applicable in very large problems if the attention is changed to find a preferred region on the Pareto-optimal front, instead of the complete front. Some such preference based EMO studies [3, 16, 22] are applied to 10 or more objectives. In certain many-objective problems, the Pareto-optimal front can be low-dimensional mainly due to the presence of redundant objectives and EMO procedures can again be effective in solving such problems [5, 20, 37]. In addition, the use of reliability based EMO [12, 19] and robust EMO [15] procedures are ready to be applied to real-world multi-objective design optimization problems. Application studies are also of interest from the point of demonstrating how an EMO procedure and a subsequent MCDM approach can be combined in an iterative manner together to solve a multi-objective optimization problem. Such efforts may lead to development of GUI-based softwares and approaches for solving the task and will require addressing other important issues such as visualization of multi-dimensional data, parallel implementation of EMO and MCDM procedures, meta-modeling approaches, and others.

Besides solving real-world multi-objective optimization problems, EMO procedures are also found to be useful for a knowledge discovery task related to a better understanding of the problem. After a set of trade-off solutions are found by an EMO, these solutions can be compared against each other to unveil interesting principles which lie common to all these solutions. These common properties among high-performing solutions will provide useful insights about what makes a solution optimal in a particular problem. Also, investigating what properties are not common may provide information about what makes the solutions to differ from each other. Such useful information mined from the obtained EMO trade-off solutions were discovered in many real-world engineering design problems in the recent past [21] and the task of unveiling useful knowledge about a problem is termed as the task of ‘innovization’ in that study. The task is similar in concept to the product platform design task which uses multiple optimizations and statistical tools [38], but provides a systematic and direct mean of discovering common principles through optimization.

6 Conclusions

This paper has provided a brief introduction and current research focus to a fast-growing field of multi-objective optimization based on evolutionary algorithms. The EMO principle of solving multi-objective optimization has been to first find a set of Pareto-optimal solutions and then choose a preferred solution. Since an EO uses a population of solutions in each iteration, EO procedures are potentially viable techniques to find and capture a number of Pareto-optimal solutions in a single simulation run.

Besides their routine applications in solving multi-objective optimization problems, EMO has spread its wings in aiding other types of optimization problems, such as single-objective constrained optimization, clustering problems etc. EMO has been used to unveil important hidden knowledge about what makes a solution optimal. EMO techniques are increasingly being found to have tremendous potential to be used in conjunction with multiple criteria decision making (MCDM) tasks in not only finding a set of optimal solutions but also to aid in selecting a preferred solution at the end. In the reference and appendix sections, we provide some useful information about the EMO research, so that interested readers can get exposed to and involved with the vast literature of EMO.

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- A. Osyczka. *Evolutionary algorithms for single and multicriteria design optimisation*, Heidelberg: Physica-Verlag, 2002.
A book describing single and multi-objective EAs with many engineering applications.
 - Y. Collette and P. Siarry. *Multiobjective Optimization: Principles and Case Studies*, Berlin: Springer, 2004.
This book describes multi-objective optimization methods including EMO and decision-making, and a number of engineering case studies.
 - N. Nedjah and L. de Macedo Mourelle (Eds.). *Real-World Multi-Objective System Engineering*, New York: Nova Science Publishers, 2005.
This edited book discusses recent developments and application of multi-objective optimization including EMO.
 - M. Sakawa. *Genetic Algorithms and Fuzzy Multiobjective Optimization*, Norwell, MA: Kluwer, 2002.
This book discusses EMO for 0-1 programming, integer programming, nonconvex programming, and job-shop scheduling problems under multiobjectiveness and fuzziness.
 - K.C. Tan, E.F. Khor, T.H. Lee. *Multiobjective Evolutionary Algorithms and Applications*, London, UK: Springer-Verlag, 2005.
A book on various methods of preference-based EMO and application case studies covering areas such as control and scheduling.

Some Review Papers

- C.A.C. Coello. *Evolutionary Multi-Objective Optimization: A Critical Review*. In R. Sarker, M. Mohamadian and X. Yao (Eds), *Evolutionary Optimization*, pp. 117–146, Kluwer Academic Publishers, New York, 2002.
- C. Dimopoulos. *A Review of Evolutionary Multiobjective Optimization Applications in the Area of Production Research. 2004 Congress on Evolutionary Computation (CEC'2004)*, IEEE Service Center, Vol. 2, pp. 1487–1494, 2004.
- S. Huband, P. Hingston, L. Barone and L. While. *A Review of Multiobjective Test Problems and a Scalable Test Problem Toolkit*, *IEEE Transactions on Evolutionary Computation*, Vol. 10, No. 5, pp. 477–506, 2006.

Dedicated Conference Proceedings

- S. Obayashi, K. Deb, C. Poloni, and T. Hiroyasu (Eds.), *Evolutionary Multi-Criterion Optimization (EMO-07) Conference Proceedings*, Also available as LNCS 4403. Berlin, Germany: Springer, 2007.
- C.A.C. Coello, A.H. Aguirre, E. Zitzler (Eds.), *Evolutionary Multi-Criterion Optimization (EMO-05) Conference Proceedings*, Also available as LNCS 3410. Berlin, Germany: Springer, 2005.
- Fonseca, C., Fleming, F., Zitzler, E., Deb, K., and Thiele, L. (Eds.), *Evolutionary Multi-Criterion Optimization (EMO-03) Conference Proceedings*. Also available as LNCS 2632. Heidelberg: Springer, 2003.

Appendix: EMO Repository

Here, we outline some dedicated literature in the area of multi-objective optimization. Further details can be found in the reference section.

Books in Print

- A. Abraham, L.C. Jain, R. Goldberg. *Evolutionary Multiobjective Optimization: Theoretical Advances and Applications*, London: Springer-Verlag, 2005.
This is a collection of the latest state-of-the-art theoretical research, design challenges and applications in the field of EMO.
- C.A.C. Coello, D.A. VanVeldhuizen, G. Lamont. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Boston, MA: Kluwer Academic Publishers, 2002.
A good reference book with a good citation of most EMO studies. A revised version is in print.
- K. Deb. *Multi-objective optimization using evolutionary algorithms*. Chichester, UK: Wiley, 2001. (Third edition, with exercise problems)
A comprehensive text-book introducing the EMO field and describing major EMO methodologies and salient research directions.

- Zitzler, E., Deb, K., Thiele, L., Coello, C. A. C. and Corne, D. (Eds.), *Evolutionary Multi-Criterion Optimization (EMO-01) Conference Proceedings*, Also available as LNCS 1993. Heidelberg: Springer, 2001.
- GECCO (LNCS series of Springer) and CEC (IEEE Press) annual conference proceedings feature numerous research papers on EMO theory, implementation, and applications.
- NSGA-II in C: <http://www.iitk.ac.in/kangal/soft.htm>
- PISA: <http://www.tik.ee.ethz.ch/sop/pisa/>
- SPEA2 in C++: <http://www.tik.ee.ethz.ch/zitzler/>
- shark in C++: <http://shark-project.sourceforge.net>
- Further information: <http://www.lania.mx/ccoello/EMO/>

Mailing lists

- emo-list@ualg.pt (EMO methodologies)
- MCRIT-L@LISTSERV.UGA.EDU (MCDM methodologies)

Public-domain source codes

- NIMBUS: <http://www.mit.jyu.fi/MCDM/soft.html>

Commercial codes implementing EMO

- iSIGHT and FIPER from Engineous (<http://www.engineous.com/>)
- GEATbx in Matlab (<http://www.geatbx.com/>)
- MAX from CENAERO (<http://www.cenaero.be/>)
- modeFRONTIER from Esteco (<http://www.esteco.com/>)